ESSAYS ON COMPETITIVE SPATIAL PRICE DISCRIMINATION

By

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To my parents

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This dissertation is a collection of three essays on spatial price discrimination.

The first essay analyzes a two-stage duopoly model. The firms choose location simultaneously in the first stage. They sell an identical good to consumers uniformly distributed along a line segment and set spatially discriminatory prices in the second stage. I show that the equilibrium locations are not the welfare maximizing locations when production costs are nonlinear. Nonlinear production costs permit analysis of symmetric reciprocal externalities. When marginal cost is affected by a rival firm's output or location choice, the Nash equilibrium locations do not maximize welfare. If only fixed costs are affected. then the welfare maximizing locations are sustained as the Nash equilibrium of the game.

The second essay considers the issue of sequential entry with deterrence in a standard spatial price discrimination model with perfect foresight. With an exogenously specified number of firms, there is an asymmetric Stackelberg location equilibrium in a duopoly. With three firms, the third firm locates in the center of its market, and the first two locate symmetrically around it. The analysis is extended to make the entry process endogenous. Depending on the number of incumbents and on the level of sunk cost, location may be used as an entry barrier.

The third essay is an extension of the sequential entry paradigm to look at entry decisions in a duopoly setup when the market can grow over time. With complete information about market type, the Nash equilibrium locations and price schedules are derived in both a one-period and a two-period model. Asymmetric information is introduced, where the informed firm signals market type through its location choice. It is shown that both in the one-period and in the two-period model, the informed firm has an incentive to locate as if the market has grown. Hence the complete information equilibrium locations cannot be sustained as a Nash equilibrium, with asymmetric information.

The study of a spatial market with varying assumptions about cost structure, entry and information, in a setup where firms can price discriminate over consumers, is the focus of this research.

CHAPTER 1
INTRODUCTION

A spatial market is one where the location of buyers and sellers of a single homogeneous good is an important variable in the determination of market equilibrium. In a pioneering paper, Hotelling (1929) provided a framework for studying spatial competition. In Hotelling's work, firms follow a policy of mill pricing. That is, consumers bear transportation costs and each firm sets a single mill price irrespective of consumer location.

This work is based on Hotelling's (1929) model of spatial competition, with the difference that here firms follow a policy of price discrimination (see Phlips, 1983). That is, firms bear transport costs and deliver their product. Each firm chooses a price schedule $p(x)$, which denotes the delivered price for a consumer located at x.

Within the framework of spatial price discrimination, each chapter investigates a specific characteristic of the market. The second chapter discusses location choice with nonlinear production costs in the context of spatial price discrimination. While spatial competition introduces an element of realism by incorporating transport costs as an important element of total costs, it has been restricted to linear production costs. One aspect of this chapter is to generalize the cost function to obtain fresh insight into the properties of equilibrium locations and to contrast them with the existing results. Then the issue of production externalities in a spatial context is discussed. Symmetric reciprocal externalities between duopolists are introduced such that production costs are affected both by the output of and by the distance from the rival firm. It is demonstrated that the issue of externalities can formally be treated within the framework of nonlinear costs of production.

In the real world, it is observed that prices are usually much more flexible than locations. Firms revise prices often (occasional sales and discounts) and relatively costlessly, in contrast to altering their locations. In this context, if firms enter an industry sequentially, the incumbent firm can only alter its price; its location is a once-and-for-all choice. The third chapter analyzes the issue of sequential entry and deterrence, with spatial price discrimination. It is well known that a location choice by an early entrant constitutes a credible commitment (see Encaoua, Geroski and Jacquemin, 1986; Gilbert, 1986). When the entry process is sequential and location is a binding commitment, it is reasonable to expect that an early entrant will anticipate future competition and incorporate this in its optimal location choice (for the fixed price case, see Prescott and Visscher, 1977). In this work, the incumbent firm(s) choose locations noncooperatively, to deter subsequent entry. The problem is first analyzed with an exogenously specified number of firms, and then extended to make the entry process endogenous.

The fourth chapter analyzes the issue of sequential entry in real time. In a setting where the market can grow in the future, a duopoly model with sequential entry is analyzed, where the Stackelberg leader has better information about the market area, relative to the follower. Although the issue of a growing market in a model of sequential entry seems relevant, most studies focus on a fixed market size (see Eaton and Lipsey, 1975; Lane, 1980; Prescott and Visscher, 1977). In this chapter. both a one-period and a two-period model are analyzed. In the one-period model, the duopolists enter the market sequentially and choose their locations, after which they simultaneously announce price schedules and the market clears. Given complete information about market type, the Nash equilibrium prices and locations are derived. Then the case of asymmetric information is analyzed where only the first entrant is informed about market type. It is shown that the complete information equilibrium locations cannot be sustained as a signalling equilibrium with asymmetric information. The analysis is extended to the two-period model. In period one, the Stackelberg leader is a monopolist. The second firm enters at the beginning of period two, and price schedules are simultaneously chosen. It is shown that when only the Stackelberg leader is informed about the market type, the complete information equilibrium locations cannot be sustained as a signalling equilibrium.

CHAPTER 2
SPATIAL PRICE DISCRIMINATION: THE EFFECT OF NONLINEAR PRODUCTION COST AND EXTERNALITIES

Section 2.1: Introduction and Literature Review

The study of spatial competition originated with Hotelling (1929). Although it has a long history, the literature to date still rests on certain simplyfying assumptions. For example, a standard assumption is constant marginal costs of production. This extremely restrictive assumption rules out several possibilities that arise in price discrimination models. One aspect of this paper is to generalize the cost function to obtain fresh insight into the properties of equilibrium strategies and to contrast them with the existing results. The other purpose of this paper is to introduce production externalities in spatial models. It will be shown that the model with production externalities can formally be treated within the framework of nonlinear costs of production.

Hotelling (1929) considered two identical firms producing a single homogeneous good, with a constant marginal cost of production. Consumers were uniformly distributed along a line which characterized the market and they had perfectly inelastic demands. Finally, he assumed that firms used mill pricing. Hotelling argued that the firms would locate at the center of the market. However profit functions are not always quasiconcave in prices, and equilibrium in pure strategies does not always exist for all locations. This was independently discovered by Vickery (1964) and d'Aspremont, Gabswewicz and Thisse (1979).[1]

One alternative to consider is spatial price discrimination. Hoover (1936) suggested that local monopoly power makes spatial price discrimination feasible. Competitive spatial price discrimination has been analyzed by Lederer and Hurter (1985, 1986), who studied discriminatory pricing in the Hotelling model. They proved the existence and analyzed the properties of equilibrium in a model of competition between firms which chose locations and then delivered price schedules. Greenhut and Greenhut (1975) discussed spatial discrimination by quantity-setting firms. Hamilton, Thisse and Weskamp (1989) compared Bertrand and Cournot equilibria in a duopoly model with linear demand and price discrimination.

The implications of letting firms choose their price policies in a spatial competition model with fixed locations for firms was analyzed in Thisse and Vives (1988). In their model, either firms simultaneously choose price policies and actual prices or firms commit to certain policies prior to the actual price competition. They concluded that, in general, there was a robust tendency for a firm to choose to discriminate.

It can be argued that industries which practice price discrimination would have certain specific characteristics. In the first place, the transport cost should be significant so that firms should not be located too close to the buyers.[2] Secondly, the firm should have some inherent advantage over the buyer, in transporting the good.[3]

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[1] Osborne and Pitchik (1987) allowed mixed strategies in the price subgame and found the subgame perfect Nash equilibrium.

This paper contributes to the existing literature in two ways: (i) a general production cost function is assumed; (ii) reciprocal externalities between the duopolists are introduced such that production costs are affected both by the output of and by the distance from the rival firm. The analysis is carried out in a two-stage game played in a duopoly setup where the two firms are costlessly mobile along a line of unit length and where they price discriminate over consumers who are uniformly distributed along this line.

The paper is organized as follows: Section 2.2 presents the basic model and the assumptions. Section 2.3 introduces production costs. Section 2.4 deals with output and location externalities. Section 2.5 is a summary of results and indicates some directions for future extensions.

Section 2.2: Essentials of The Model

In this model we consider a two-stage game played by two firms, denoted 1 and 2, which produce a single homogenous output q. In stage 1.

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[2] This is relevant if we consider the case of a monopolist who price discriminates over buyers according to their locations, such that differences in prices charged, reflects more than the difference in transport cost.

[3] Industries like iron and steel, cement or oil fit the above conditions. A study of spatial pricing patterns in the U.S. (Greenhut. Greenhut and Li, 1980) showed that pricing patterns of firms were largely determined by locational patterns of competitors as well as varying intensity of competition at different spatial market points. Their study concluded that price discrimination over geographic space was the most prevalent pricing technique.

each firm simultaneously chooses a location. In stage 2, each observes its competitor's location and chooses a price schedule.

Let a and b denote the distances of firm 1 and 2 from the left and right endpoints, respectively, of a bounded market represented by a line of unit length. Let $P_i(x, a, b)$ denote the price schedule offered by firm i to a consumer located at x, where x is the distance from the left end point of the market.

Assumptions

A1 There is a uniform distribution of consumers over the market.

A2 Each consumer has a perfectly inelastic demand for one unit of the commodity.

A3 If a consumer faces the same delivered price from both firms, he buys the commodity from the nearer firm.[4]

A4 Transport cost is a linear function of the distance between the firm and the consumer.

A5 $C_i(q_i, q_j, d)$ represents the cost function of a firm. In section 2.3, $C(q_i)$ is a function of own firm's output. In section 2.4.1, $C_i(q_i, q_j)$, $i \neq j$, $i, j = 1, 2$ depends on both q_i and q_j. This is assumed to be due to an externality generated by rival firm's output. In section 2.4.2, $C_i(d)$ is a function of the distance from the rival firm, represented by d. Also the case of $C_i(d, q_i)$ is considered where the distance between the firms affects marginal costs.

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[4] If another assignment rule is used when firms charge identical prices, the nearer firm can reduce its price by ε and secure the corresponding markets.

Our attention is restricted to subgame perfect Nash equilibria in location and price schedules. A price schedule (P_1*, P_2*) is a Nash equilibrium, given locations (a, b) if and only if $\pi_1*(P_1*, P_2*) \geq \pi_1(P_1, P_2*)$ and $\pi_2*(P_1*, P_2*) \geq \pi_2*(P_1*, P_2)$. Thus for each (a, b), we can solve for the optimal price schedules (P_1^*, P_2^*) and express these as functions of (a, b). We then substitute these into the profit functions and express the profit of each firm as a function of locations (a, b). A pair of locations (a*, b*) forms a Nash equilibrium if and only if $\pi_1(P_1*(a*, b*), P_2*(a*, b*), a*, b*) \geq \pi_1(P_1(a, b*), P_2(a, b*), a, b*) \; \forall \; a$ and $\pi_2(P_1*(a*, b*), P_2*(a*, b*), a*, b*) \geq \pi_2(P_1(a*, b), P_2(a*, b), a*, b) \; \forall \; b$. In the model, the welfare maximizing locations of the two firms (a_w, b_w) are characterized by minimum delivered cost; that is, the locations at which the two firms minimize the sum of production and transport cost.

Section 2.3: Effect of Production Costs in the Spatial Price Discrimination Model

In this section, we analyze the effect of nonlinear production costs on the Nash equilibrium price and locations of the two players. In the literature, the standard assumption is constant marginal costs of production. Here we consider a general cost function to analyze the effect of strictly convex or strictly concave production costs on equilibrium.

We derive the symmetric Nash price and location equilibria and analyze the role of local convexity/concavity on the location equilibrium.

Let $C(q_i) = z_1 q_i + z_2 E(q_i) \quad i = 1, 2$ ...(2.1) be the production cost function of firm i, where $z_1 > 0$, $z_2 \geq 0$ and $E(q_i)$ is a function of output.

In this specification of costs, when $z_2 = 0$, we have the standard case of linear production costs. When $z_2 > 0$, then $C(q_i)$ is convex/concave according as $E''(q_i) \gtrless 0$. So, we can focus on the effect of strictly convex/concave costs by suitably taking the sign of the second derivative of $E(q_i)$. Define $\bar{x}$ as the consumer location where delivered marginal cost is equal for both firms. That is,

$c(\bar{x} - a) + z_1 + z_2E'(q_1) = c(1 - b - \bar{x}) + z_1 + z_2E'(q_2)$

$\Rightarrow \bar{x} = (1 + a - b)/2 + [z_2\{E'(q_2) - E'(q_1)\}]/2c$

Using assumption A3, it is clear that $\bar{x}$ forms a market boundary for each firm. That is, with inelastic demands, $q_1 = \bar{x}$, $q_2 = 1 - \bar{x}$.

For subgame perfect Nash equilibria, we use backward induction and derive stage 2 price schedules first. Then we determine the equilibrium location decisions taken in stage 1 of play. In general, a symmetric subgame perfect Nash equilibrium in pure strategies may not exist for a concave cost function. That is because, in equilibrium, neither firm can benefit from lowering the prices it charges to the customers currently served by the other firm. If its cost function is concave, an output expansion reduces its marginal cost and may thus increase its profit. However restrictions on parameters can be imposed to ensure existence.

Proposition 2.1

If there exists a subgame perfect Nash equilibrium, the equilibrium price schedule is such that

$P_1^*(x, a, b) = P_2^*(x, a, b)$

$= \max[c|x - a| + z_1 + z_2 \cdot E'(x), c|1 - b - x| + z_1 + z_2 \cdot E'(1 - x)]$

Proof: The usual argument for Bertrand competition implies that in equilibrium, both firms will offer equal prices to a consumer located at x.

$\Rightarrow P_1^*(x, a, b) = P_2^*(x, a, b)$.

In equilibrium each firm offers the consumer a price which is the maximum of its own delivered marginal cost and its competitor's delivered marginal cost. That is,

$P_1^*(x, a, b) = P_2^*(x, a, b,)$

$= \max[c|x - a| + z_1 + z_2E_1', c|1 - b -x| + z_1 + z_2E_2']$.

where c = transportation rate per unit distance

$|x - a|$, $|1 - b -x|$ = distance between consumer located at x and firm 1 and firm 2 located at a and b, respectively.

$z_1 + z_2E_i'$ = marginal cost of production of firm i. (i = 1, 2). ▲

Under Bertrand competition, equilibrium delivered price schedules of both firms are equal, given a and b. Further from assumption A3. each consumer buys from the firm located closer to him, given that he faces equal delivered prices from both firms. Hence, it follows that each firm can charge a price that is the maximum of both firms' delivered marginal costs. The possibility of a monopoly price lower than a rival's cost is ruled out by the assumption of perfectly inelastic demand in a Bertrand game.

Anticipating equilibrium price schedules, the firms first simultaneously choose their profit maximizing location. In Lederer and Hurter (1986), the standard assumption of constant marginal production costs leads to a symmetric location equilibrium at the quartiles of the market. At this point, we introduce concavity or convexity in production cost to perturb this equilibrium. A strict convexity in the cost function induces each firm to disperse to a location outside the quartiles, while strict concavity of costs leads to a movement towards the center of the market. Thus, we obtain a local result that strict convexity or concavity of production costs yields a definite pattern of movement away from the quartiles. This result is summarized in the proposition below:

Proposition 2.2

> The symmetric Nash location equilibrium of the two firms will locally differ from the quartiles, such that $a^*, b^* \gtrless 1/4$ according to whether the cost function is locally strictly concave/convex.

Proof: The profit functions of firm 1 and 2 are:

$$\pi_1(a,b) = \int_0^{\bar{x}} [P_1{*}(x,\ a,\ b) - c|x - a|]\ dx - C_1(q_1) \qquad \ldots(2.2)$$

$$\pi_2(a,b) = \int_{\bar{x}}^{1} [P_2{*}(x,\ a,\ b) - c|1 - b - x|]dx - C_2(q_2) \qquad \ldots(2.3)$$

where $\bar{x}$ is the consumer location such that the delivered marginal cost is equal for both firms (i.e. $c(\bar{x} - a) + z_1 + z_2E_1' = c(1-b-\bar{x}) + z_1 + z_2E_2'$). where the LHS term is the delivered marginal cost of firm 1 and the RHS term is the delivered marginal cost of firm 2. Therefore,

$$\bar{x} = \frac{1 + a - b}{2} + \frac{z_2(E'(q_2) - E'(q_1))}{2c} \qquad \text{...(2.4)}$$

Consider profit maximization by firm 1. From (2.2)

$$\underset{a}{\text{Max}}\ \pi_1(a, b) = \int_0^{\bar{x}} [P_1^*(x, a, b) - c|x - a|]\ dx - C_1(q_1)$$

Using (2.1) and Proposition 2.1, we have

$$\pi_1(a, b) = \int_0^{\bar{x}} [c(1 - b - x) + z_1 + z_2E_2' - c|x - a|]\ dx - [z_1q_1 + z_2E(q_1)].$$

$$\text{or, } \pi_1(a,b) = \int_0^{\bar{x}} c(1 - b - x) - c|x - a|]\ dx + z_1\int_0^{\bar{x}} x'\ dx' + z_2 \int_0^{\bar{x}} E_2'dx$$
$$- [z_1q_1 + z_2E(q_1)] \qquad \text{...(2.5)}$$

Now, from the definition of $\bar{x}$, it forms a market boundary for each firm, such that

$$q_1 = \int_0^{\bar{x}} x'dx' \text{ and } q_2 = \int_{\bar{x}}^{1} x'dx'$$

Using (2.4),

$$\left.\begin{aligned} q_1 &= \frac{1 + a - b}{2} + \frac{z_2}{2c}(E'(q_1) - E'(q_2)) \\ q_2 &= \frac{1 - a + b}{2} + \frac{z_2}{2c}(E'(q_1) - E'(q_2)) \end{aligned}\right\} \qquad \text{...(2.6)}$$

Therefore from (2.5),

$$\pi_1(a, b) = \int_0^{\bar{x}} [c(1 - b - x) - c|x - a|]\ dx + z_1\bar{x} + z_2E'(q_2)\bar{x} - z_1\bar{x} - z_2E(q_1)$$

The first-order condition implies

$$\frac{\partial \pi_1(a,\, b)}{\partial a} = -2ac + c\bar{x} + c[1 - a + b - 2\bar{x}]\,\frac{\partial \bar{x}}{\partial a} + z_2[E''(q_2)\bar{x}\,\frac{\partial q_2}{\partial a} + E'(q_2)\frac{\partial \bar{x}}{\partial a}$$

$$-z_2E'(q_1)\frac{\partial q_1}{\partial a}\ \ ] = 0$$

$$\text{or, } a^* = \tfrac{1}{2}\,\bar{x} + \tfrac{1}{2}\,[1 - a + b - 2\bar{x}]\,\frac{\partial \bar{x}}{\partial a} + (1/2c)z_2[E''(q_2)\bar{x}\,\frac{\partial q_2}{\partial a} + E'(q_2)\,\frac{\partial \bar{x}}{\partial a}]$$

$$-\tfrac{1}{2}\,z_2E'(q_1)\,\frac{\partial q_1}{\partial a} \qquad \ldots(2.7)$$

$\bar{x}$ (in symmetric equilibrium) = ½

$$\frac{\partial \bar{x}}{\partial a} = \tfrac{1}{2} + \frac{z_2}{2c}\,[E''(q_2)\,\frac{\partial q_2}{\partial a} - E''(q_1)\,\frac{\partial q_1}{\partial a}\ ] \qquad \ldots(2.8)$$

From (2.6):

$$\left.\begin{aligned} \frac{\partial q_1}{\partial a} &= \tfrac{1}{2} + \frac{z_2}{2c}\,[E''(q_2)\frac{\partial q_2}{\partial a} - E''(q_1)\frac{\partial q_1}{\partial a}\ ] \\ \frac{\partial q_2}{\partial a} &= -\tfrac{1}{2} - \frac{z_2}{2c}\,[E''(q_2)\,\frac{\partial q_2}{\partial a} - E''(q_1)\frac{\partial q_1}{\partial a}\ ] \end{aligned}\right\} \qquad \ldots(2.9)$$

Now (2.9) => $\dfrac{\partial q_2}{\partial a} = -\dfrac{\partial q_1}{\partial a}$

Using (2.9) in (2.8),

$$\Rightarrow \frac{\partial \bar{x}}{\partial a} = \tfrac{1}{2} - \frac{z_2}{2c}\,[E''(q_2) + E''(q_1)]\frac{\partial q_1}{\partial a} \qquad \ldots(2.10)$$

Also, from (2.9) we have

$$\frac{\partial q_1}{\partial a} = \tfrac{1}{2} - \frac{z_2}{2c}\,[E''(q_2) + E''(q_1)]\frac{\partial q_1}{\partial a}$$

$$\Rightarrow \frac{\partial q_1}{\partial a} = [\frac{c}{2c + z_2(E''(q_2) + E''(q_1))}\ ] \qquad \ldots(2.11)$$

Substituting (2.11) in (2.10), we have

$$\frac{\partial \bar{x}}{\partial a} = \tfrac{1}{2} - \frac{z_2}{2c}\ [E''(q_2) + E''(q_1)][\frac{c}{2c + z_2((E''(q_2) + E''(q_1))}] \quad \ldots(2.12)$$

Finally, using (2.8), (2.9), (2.10) and (2.12) in (2.7), we have,

$$a^* = \tfrac{1}{2} + \frac{z_2}{2c}\ E'(q_2)[\tfrac{1}{2} - \frac{z_2}{2c}\ \{E''(q_2) + E''(q_1)\}\{\frac{c}{2c + z_2(E''(q_2) + E''(q_1)}\}]$$

$$- \frac{z_2}{2c}[\frac{c}{2c + z_2(E''(q_2) + E''(q_1))}][\frac{E''(q_2)}{2} + E'(q_1)] \quad \ldots(2.13)$$

From (2.13),

$$\frac{\partial a^*}{\partial z_2} = \frac{E'(q_2)}{4c} - \frac{[\{2c + z_2(E''(q_2) + E''(q_1))\}2z_2 - z_2^2(E''(q_2) + E''(q_1))]}{[2c + z_2(E''(q_2) + E''(q_1))]^2} \ .$$

$$\frac{[E''(q_2) + E''(q_1)]}{4c} - \frac{[\{2c + z_2(E''(q_2) + E''(q_1))\} - z_2\{E''(q_2) + E''(q_1)\}]}{[2c + z_2(E''(q_2) + E''(q_1))]^2}$$

$$\frac{.[E''(q_2) + 2E'(q_1])}{4}$$

$$\Rightarrow \left.\frac{\partial a^*}{\partial z_2}\right|_{z_2 = 0} = \frac{E'(q_2)}{4c} - \frac{c[E''(q_2) + 2E'(q_1)]}{2(2c)^2}$$

$$= - \frac{1}{8c}\ E''(q_2) - \frac{1}{4c}[E'(q_1) - E'(q_2)]$$

$$\Rightarrow \left.\frac{\partial a^*}{\partial z_2}\right|_{z_2 = 0} = - \frac{1}{8c}\ E''(q_2) \quad [E'(q_1) = E'(q_2) \text{ in a symmetric equilibrium}]$$

From the above,

$$\Rightarrow \left.\frac{\partial a^*}{\partial z_2}\right|_{z_2 = 0} \ {}^{>}_{<}\ 0 \text{ according to whether } E''(q_2)\ {}^{<}_{>}\ 0 \quad \ldots(2.14)$$

The result follows, by symmetry, for π_2* and b^*. ▲

From (2.13) it is clear that at $z_2 = 0$, $a^* = \tfrac{1}{4}$. This is the standard result of equilibrium location at the quartiles with constant marginal cost of production. Then the above result in (2.14) shows that, as we increase z_2 slightly from zero, around this equilibrium, a small convexity ($E'' > 0$) or concavity ($E'' < 0$) clearly determines the directions in which equilibrium locations of the firms will change. Recall that in the constant marginal cost case, the incentive to move (at the margin) is only to lower transport costs. This is because there are no gains to obtaining the new marginal customer (due to Bertrand competition), and the prices to original customers are unchanged because they equal the rival's delivered price. Here, as the firm moves, the delivered price of its rival changes since the latter sells more or less (depending on direction of movement) and has nonlinear total production costs. The incentive at the quartiles is to move in the direction which will raise the rival's marginal cost. For instance, if marginal costs are strictly increasing, then by moving away (outside the quartiles) the rival firm gains market share and incurs higher marginal cost, thus making it possible for the original firm to charge all its retained customers a higher price. Analogous reasoning applies to the case of strictly decreasing marginal costs.

This result is a local result; in the neighborhood of equilibrium with a linear production function, we introduce a slight convexity or concavity and derive the resulting pattern of movement toward the new equilibrium. Nevertheless, it is important in that it clearly indicates the expected results with nonconstant marginal production costs. The following example illustrates the global validity of this result.

Example

Consider a quadratic cost function of the form:

$c(q_i) = \alpha + \beta q_i + \delta(q_i)^2;\ i = 1,\ 2; \qquad \alpha,\ \beta > 0;\ \delta \gtrless 0$

where $c''(q_i) = 2\delta$. So $\delta > 0$ for a strictly convex cost function while $\delta < 0$ for a strictly concave cost function. In the earlier analysis an arbitrarily small value of δ, around $\delta = 0$ was used. Here, we consider all $\delta \gtrless 0$ and check to see whether the aforementioned results are globally true. It can be shown that if a subgame perfect Nash equilibrium exists, the price schedules $(P_1*,\ P_2*)$ will be such that

$P_1*(x,\ a,\ b) = P_2*(x,\ a,\ b) = \max\ [c|x - a| + \beta + 2\delta q_1,$

$c|1 - b - x| + \beta + 2\delta q_2]$. The unique symmetric Nash location equilibria $(a*,\ b*)$ will be such that $a* = b* = \frac{1}{4}[(c + \delta)/(c + 2\delta)]$.[5]

Proof: Each firm chooses profit maximizing locations, having anticipated equilibrium price schedules.

The profit functions for firm 1 and 2 are

$$\pi_1^*(a,\ b) = \int_0^{\bar{x}} [P_1^*(x,\ a,\ b) - c|x - a|]dx - c(q_1) \qquad \ldots(2.15)$$

$$\pi_2^*(a,\ b) = \int_{\bar{x}}^{1} [P_2^*(x,\ a,\ b) - c|1 - b - x|]dx - c(q_2) \qquad \ldots(2.16)$$

where $\bar{x}$ is the consumer location where the delivered price is equal for both firms. That is,

$P_1(\bar{x},\ a,\ b) = P_2(\bar{x},\ a,\ b)$

---

[5] For a concave cost function, $0 > \delta > -c/4$ and α sufficiently small are sufficient to ensure existence. For a convex cost function, we need α sufficiently small.

$\Rightarrow c(\bar{x} - a) + \beta + 2\delta q_1 = c(1 - b - \bar{x}) + \beta + 2\delta q_2$

or, $2c\bar{x} = 2\delta(q_2 - q_1) + c(1 + a - b)$

or, $\bar{x} = \dfrac{\delta(q_2 - q_1)}{c} + \dfrac{(1 + a - b)}{2}$ ...(2.17)

Now, $q_1 = \int_0^{\bar{x}} x'dx'$

$q_2 = \int_{\bar{x}}^{1} x'dx'$.

$\Rightarrow q_1 = \bar{x},\ q_2 = 1 - \bar{x}$ ...(2.18)

Substituting q_1 from (2.18) in (2.17)

$q_1 = \dfrac{\delta(q_2 - q_1)}{c} + \dfrac{(1 + a - b)}{2}$ ...(2.19)

or, $[1 + \dfrac{\delta}{c}]q_1 = \dfrac{\delta}{c} q_2 + \dfrac{1 + a - b}{2}$ ...(2.20)

Similarly,

$[1 + \dfrac{\delta}{c}]q_2 = \dfrac{\delta}{c} q_1 + \dfrac{1 + a - b}{2}$ ...(2.21)

Adding (2.20) and (2.21) $\Rightarrow q_1 + q_2 = 1$ ...(2.22)

Substituting (2.22) in (2.19)

$q_1 = \dfrac{\delta(1 - 2q_1)}{c} + \dfrac{(1 + a - b)}{2}$

or $q_1 = \dfrac{\delta}{c} - \dfrac{\delta}{c} q_1 + \dfrac{(1 + a - b)}{2}$

or $[1 + \dfrac{2\delta}{c}]q_1 = \dfrac{\delta}{c} + \dfrac{(1 + a - b)}{2}$

$$\text{or } [c + 2\delta]q_1 = \frac{2\delta + (1 + a - b)c}{2}$$

$$\text{or } q_1 = \frac{(c + 2\delta) + c(a - b)}{2(c + 2\delta)}$$

$$\text{or } q_1 = \tfrac{1}{2} + \frac{c(a - b)}{2(c + 2\delta)} \qquad \ldots(2.23)$$

$$\text{Similarly, } q_2 = \tfrac{1}{2} + \frac{c(b - a)}{2(c + 2\delta)} \qquad \ldots(2.24)$$

Substituting (2.23) and (2.24) in (2.17)

$$\bar{x} = \frac{\delta(b - a)}{c + 2\delta} + \frac{(1 - b + a)}{2}$$

$$\text{or, } \bar{x} = \tfrac{1}{2} - \frac{c(b - a)}{2(c + 2\delta)} \qquad \ldots(2.25)$$

Now, using P_1^*, P_2^* and $c(q_i)$ in (2.15) and (2.16),

$$\pi_1^*(a, b) = \int_0^{\bar{x}} [c(1 - b - x) + \beta + 2\delta q_2 - c|x - a|]dx - [\alpha + \beta q_1 + \delta q_1^2] \qquad \ldots(2.26)$$

$$\pi_2^*(a, b) = \int_{\bar{x}}^1 [c(x - a) + \beta + 2\delta q_1 - c|1 - b - x|]dx - [\alpha + \beta q_2 + \delta q_2^2] \qquad \ldots(2.27)$$

Consider the profit of firm 1. Using (2.18) in (2.26), we get

$$\pi_1^*(a, b) = \int_0^{\bar{x}} [c(1 - b - x) + \beta + 2\delta\int_{\bar{x}}^1 x'dx' - c|x - a|]dx - [\alpha + \beta\int_0^{\bar{x}} x'dx' + \delta(\int_0^{\bar{x}} dx)^2] \qquad \ldots(2.28)$$

The first-order condition implies

$$\frac{\partial \pi_1}{\partial a} = -c\int_0^a x'dx' + c\int_a^{\bar{x}} x'dx' + c[1 + a - b - 2\bar{x}]\frac{\partial \bar{x}}{\partial a} + 2\delta\frac{\partial \bar{x}}{\partial a} - 6\delta\bar{x}\frac{\partial \bar{x}}{\partial a} = 0$$

$$\text{or, } -2ac + c\bar{x} + c[1 + a - b - 2\bar{x}]\frac{\partial \bar{x}}{\partial a} + 2\delta\frac{\partial \bar{x}}{\partial a} - 6\delta\bar{x}\frac{\partial \bar{x}}{\partial a} = 0 \quad \ldots(2.29)$$

From (2.25) we know

$$\left.\begin{array}{l} \bar{x} \text{ (in equilibrium)} = \frac{1}{2} \\ \frac{\partial \bar{x}}{\partial a} = \frac{1}{2}\left[\frac{c}{c + 2\delta}\right] \end{array}\right\} \quad \ldots(2.30)$$

Substituting (2.30) in (2.29) we get

$$2a^* = \frac{1}{2} + 2\delta\left[\frac{1}{2(c + 2\delta)}\right] - 3\delta\left[\frac{1}{2(c + 2\delta)}\right]$$

$$\text{or, } a^* = \frac{1}{4} + \frac{\delta}{2(c + 2\delta)} - \frac{3\delta}{4(c + 2\delta)}$$

Simplyfying, we obtain

$$a^* = \frac{1}{4}\left[\frac{c + \delta}{c + 2\delta}\right].$$

By symmetry,

$$b^* = a^* = \frac{1}{4}\left[\frac{c + \delta}{c + 2\delta}\right] \gtrless \frac{1}{4} \text{ according to whether } \delta \lessgtr 0.$$

The equilibrium is unique since the reaction functions are increasing in a and b and one is always steeper than the other. Second-order conditions for profit maximization imply

$$\frac{\partial^2 \pi_1}{\partial a^2} = -2c + c\frac{\partial \bar{x}}{\partial a} + c\left[1 - 2\frac{\partial \bar{x}}{\partial a}\right]\frac{\partial \bar{x}}{\partial a} - 6\delta\bar{x}\left(\frac{\partial \bar{x}}{\partial a}\right)^2 = 0$$

Substituting (2.30) and simplyfying,

$$\frac{\partial^2\pi_1}{\partial a^2} = -c\left[\frac{3c^2 + 15\delta c + 16\delta^2}{2(c + 2\delta)}\right] < 0 \quad \forall\ \delta > 0,\ -c/4 < \delta < 0. \qquad \blacktriangle$$

Clearly, $a^* = b^* > \frac{1}{4}$ if $\delta < 0$ and $a^* = b^* < \frac{1}{4}$ if $\delta > 0$. Note that if we set $\delta = 0$, then $a^* = b^* = \frac{1}{4}$. This is the standard case of constant marginal cost, where it is optimal for each firm to locate at the quartiles.

We now turn to the welfare properties of the equilibrium. In the standard case of zero (or constant) marginal costs, recall that both firms clearly have an incentive to locate so as to minimize transport costs, given the competing firm's location. So the profit maximizing Nash equilibria lie at the quartiles of the market. This implies that, with constant marginal costs of production, the profit maximizing location coincides with the welfare maximizing location. Proposition 2.3 shows that when production costs are strictly concave or convex, the profit maximizing Nash equilibrium locations no longer maximize welfare. To prove Proposition 2.3, Lemma 2.1 is used.

Lemma 2.1

If $z_2\{E(1) - 2E(\frac{1}{2})\} + c/8 > 0$, then the welfare maximizing locations are at the first and third quartiles of the market.

Proof: Total Cost = (Production Cost of firms 1 and 2) + (transport cost of firms 1 and 2.) The production cost function is $C_i(q_i) = z_1q_i + z_2E(q_i)$ $\forall\ i = 1, 2$ from (2.1) and $c|x-a|$, $c|1-b-x|$ are the transport costs incurred by firms 1 and 2 respectively, in serving a customer located at x.

Note that if only one firm serves the entire market, it would locate efficiently at the center of the market. Let TC(1) be the total cost of the monopoly. Then $TC(1) = z_1 + z_2E(1) + c/4$. The total cost with a duopoly when firms locate efficiently is $TC(2) = z_1 + 2z_2E(½) + c/8$. Clearly, transport cost is lower with a duopoly.

Moreover, if the cost function is convex, $TC(1) - TC(2) = z_2\{E(1) - 2E(½)\} + c/8 > 0$. However, when the cost function is concave, since $\{E(1) - 2E(½)\} < 0$, we need to restrict the parameters of the cost function to ensure that the above inequality is satisfied. This will ensure that it is more efficient to have two firms, rather than one, serving the market.

Let TC(W) represent total cost under welfare maximizing location a_w.

$$TC(W) = \sum_{i=1}^{2} [z_1q_i + z_2E(q_i)] + 2\int_0^{½} c\,|x-a|\,dx$$

$$= \sum_{i=1}^{2} [z_1q_i + z_2E(q_i)] + 2c[a^2 - ½ + 1/8]$$

The first-order condition implies

$$\frac{\partial(TC(W))}{\partial a} = \sum_{i=1}^{2}[z_1 \frac{\partial q_1}{\partial a} + z_2E'(q_i)\frac{\partial q_i}{\partial a}] + 2c[2a - ½] = 0$$

We know $\frac{\partial q_1}{\partial a} = - \frac{\partial q_2}{\partial a}$

$$\Rightarrow \frac{\partial(TC(W))}{\partial a} = 2c[2a_w - ½] = 0 \quad \Rightarrow a_w = ¼.$$ By symmetry, $b_w = ¼$.

Since a_w, b_w are independent of the parameters of the cost function, it follows that the welfare maximizing locations are independent of production costs. ▲

The above lemma says that if it is optimal to have two firms instead of a monopoly, then the welfare maximizing locations are at the quartiles. Clearly if the cost function is convex, the above inequality is always satisfied. Transport cost in duopoly is always less than with a monopoly. Furthermore, if the production cost function is convex, the above expression is always satisfied, since $E(1) > 2E(\frac{1}{2})$. However, if the cost function is concave, then we need restrictions to ensure that the above is true, since $E(1) < 2E(\frac{1}{2})$. Proposition 2.3 now follows directly from Proposition 2.2 and Lemma 2.1.

Proposition 2.3

When production costs are nonlinear, the profit maximizing locations no longer coincide with their corresponding welfare maxima.

In conclusion, the effect of nonlinear production costs on Nash equilibrium locations in a spatial framework is characterized by definite patterns of movement of the equilibria depending on the shape of the cost function and also by the phenomenon that the Nash equilibrium no longer maximizes welfare. In the next section, the analysis is extended to a case where spatially separated firms generate reciprocal negative externalities.

Section 2.4: Effect of Externalities in the Spatial Price Discrimination Model

In this section, we introduce symmetric, reciprocal externalities created by each firm on its competitor. Production externalities can certainly arise in a spatial model. (Examples are industrial air and water pollution.) This section will illustrate that we can formally treat the problem of externalities within the framework developed in the previous section. First in subsection 2.4.1, we consider an externality created through the rival firm's output level. Then in 2.4.2, the distance between the two firms determines the extent of damage.

Section 2.4.1: An Output Externality

Production externalities are created via the rival firm's output decision. Let $C_i = C_i(q_i, q_j)$, $i \neq j$; $i, j = 1, 2$. Also, $\partial C_i/\partial q_i, \partial C_i/\partial q_j > 0$. Hence, by assumption, firm j creates a negative externality on firm i. In equilibrium, given that the market is of unit length, it implies that $q_i + q_j = 1$, $i \neq j$; $i, j = 1, 2$. This relationship may be used to derive the nature of the cost function around equilibrium. This implies that $C_i(q_i, q_j) = C_i(q_i, 1 - q_i)$. In other words, the externality issue can be reduced to a problem where each firm's cost is essentially a function of its own output level. It can be shown that

$$C_i''(q_i, q_j) = \partial^2 C_i\,(q_i, q_j)/\partial q_i^2 - 2\partial^2 C_i(q_i, q_j)/\partial q_i \partial q_j + \partial^2 C_i(q_i, q_j)/\partial q_j^2 \quad \ldots(2.31)$$

where the first term on the right hand side refers to the concavity/convexity of the cost function of firm i with respect to its own output level. The second term is the change in marginal cost when q_j increases while the third term is the rate of change of marginal damage. Notice that with a negative externality, it is reasonable to assume that the second term, $\partial(\partial C_i/\partial q_i)/\partial q_j$, is nonnegative. We know from Proposition 2.2 that the Nash equilibrium locations will now depend on the sign of $C_i''(q_i, q_j)$. We will analyze this issue in two cases:

Case 1: $\partial^2 C_i(q_i, q_j)/\partial q_j^2 = 0$. This is the case of constant (direct) marginal damage. Clearly with a negative externality, if $\partial^2 C_i(q_i, q_j)/\partial q_i^2 < 0$, then $C_i''(q_i, q_j) < 0$. That is, if costs are strictly concave in own output, that is sufficient for the total cost function to be strictly concave. From Proposition 2.2 we know that the Nash location equilibrium will lie inside the quartiles. If $\partial^2C_i(q_i, q_j)/\partial q_i^2 > 0$, then the sign of $C_i''(q_i, q_j)$ is ambiguous in general and would depend on the specific functional form. Both these results are perfectly consistent with the expected behavior of the firms. When cost is strictly concave in own output $[\partial^2C_i(q_i, q_j)/\partial q_i^2 < 0]$, recall that firm i will have an incentive to move inward. This effect is reinforced by the phenomenon of a negative externality, where rising marginal costs with respect to rival's output induce the firms to move inside the quartiles. Both these effects are therefore reinforcing each other and the total effect is unambiguous. From this point it is obvious that strict convexity of costs in own output and the effect of a negative externality, in fact, act in opposite directions. Hence, not much can be said in general on the equilibrium locations. Note that if there is a positive externality $[\partial\{\partial C_i(q_i, q_j)/\partial q_i\}/\partial q_j < 0]$, then the above results are exactly reversed. Strict convexity of cost in own output combined with a positive externality gives an unambiguous direction; ambiguity arises with strict concavity and a positive externality.

Case 2: $\partial^2C_i(q_i, q_j)/\partial q_j^2 \neq 0$. This is the case where marginal (direct) damage is not constant. Clearly, we cannot say much about the sign of $C_i''(q_i, q_j)$ in this context without further specification of costs. For instance if

$$C_i(q_i, q_j) = E_i(q_i)E_j(q_j), \ i \neq j; \ i, j = 1, 2. \qquad \ldots(2.32)$$

It can be shown that

$$C_i''(q_i, q_j) = E_i''(q_i)E_j(q_j) - 2E_i'(q_i)E_j'(q_j) + E_i(q_i)E_j''(q_j) \qquad \ldots(2.33)$$

From (2.33) it is evident that if $E_i(q_i)$ and $E_j(q_j)$ are strictly concave functions ($E_i''(q_i)$, $E_j''(q_j) < 0$), then $C_i''(q_i, q_j) < 0$. Thus, strict concavity of $E_i(q_i)$ and $E_j(q_j)$ is sufficient for $C_i(q_i, q_j)$ to be strictly concave in q_i. Then, we can apply the result obtained in Proposition 2.2 to conclude that the equilibrium locations of the two firms would lie inside the quartiles. However, from (2.33), observe that if either $E_i(q_i)$ or $E_j(q_j)$ is strictly convex, that may once again give an ambiguous sign of $C_i''(q_i, q_j)$. So, with separable cost functions, we can partially answer the question about Nash equilibrium locations vis-a-vis the quartiles, when marginal damages are not constant. When E_i and E_j are both strictly concave functions, they both reinforce the movement of equilibrium location inside the quartiles. The firm tries to take advantage of decreasing costs by moving inside the quartiles. At the same time, the negative externality induces it to restrict its rival's output by expanding its own market. Both these effects reinforce each other, and the resulting effect is unambiguous.

Section 2.4.2: An Externality in Distance

In this section, it is first assumed that a negative externality arises through the distance between the two firms, such that each firm's cost is a decreasing function of this distance, d.

Let $C_i^d(d) = 1 - d, \qquad i = 1, 2.$ ...(2.34)

where $d = 1 - a - b$. Let P_i^{d*} denote the equilibrium price schedule for firm i and a^{d*}, b^{d*} denote the optimal locations. Solving by backward induction for the subgame perfect price and location equilibria, we obtain the results specified in Propositions 2.4 and 2.5 below.

Proposition 2.4

There exists a unique equilibrium delivered price schedule such that $P_1^{d}*(x, a, b) = P_2^{d}*(x, a, b) = \max[c|x - a|, c|1 - b - x|]$.

Proof: Since marginal cost is zero, the equilibrium price schedule offered by a firm will be the maximum of the two firms' transport cost.

The location equilibrium of the two firms is summarized in the proposition below.

Proposition 2.5

If the negative externalities depend on the distance between the two firms, then each firm moves outside the quartiles in a symmetric Nash equilibrium.

Proof: Let a^{d*} and b^{d*} denote the equilibrium locations.

$$\pi_1^{d}*(a, b) = \int_0^{\bar{x}} [p_1^{d}* - c|x - a|]\, dx - c_1^{d}(q_1, d) \qquad \ldots(2.35)$$

$$\pi_2^{d}*(a, b) = \int_{\bar{x}}^{1} [p_2^{d}* - c|1 - b - x|]\, dx - c_2^{d}(q_2, d) \qquad \ldots(2.36)$$

where $$\bar{x} = \frac{1 + a - b}{2} \qquad \ldots(2.37)$$

Using Proposition 2.4 and (2.37) we can solve (2.35) for a^{d*}. We have

$$\pi_1^{d}*(a, b) = \int_0^{\bar{x}} [c(1 - b - x) - c|x - a|]\, dx - (a + b)$$

The first-order condition implies

$$\frac{\partial \pi_1^{d*}(a, b)}{\partial a} = -2ac + c\bar{x} + c[1 + a - b - 2\bar{x}]\ \frac{\partial \bar{x}}{\partial a} - 1 = 0$$

or, $a^{d*} = \frac{1}{4} - (\frac{1}{2})c$

or, $a^{d*} = \dfrac{c - 2}{4c} < \frac{1}{4}$ assuming $c > 2$.

Second-order conditions

$$\frac{\partial^2 \pi_1^{d*}(a,\ b)}{\partial a^2} = -2c + c\,\frac{\partial \bar{x}}{\partial a} + c\left[1 - 2\,\frac{\partial \bar{x}}{\partial a}\right]\,\frac{\partial \bar{x}}{\partial a}$$

$$\frac{\partial^2 \pi_1^{d*}(a,\ b)}{\partial a^2} = -2c + \tfrac{1}{2}c = -(3/2)c < 0.$$ ▲

Clearly, a negative externality, being an inverse function of the distance between the firms, leads to an equilibrium such that the firms disperse away from the quartiles. The welfare aspect of this equilibrium is summarized in Proposition 2.6 below.

Proposition 2.6

> If the negative externality is a function of distance between the firms, the private optimum location is also the welfare maximizing location.

Proof: Total Cost = production cost + transport cost

$$= C_1^d(d) + C_2^d(d) + \int_0^{\frac{1}{2}} c\,|x - a|\,dx$$

From (2.34),

$$\text{Total cost} = 2(a + b) + 2\int_0^{\frac{1}{2}} c\,|x - a|\,dx$$

The first-order condition implies

$$c[2a - \tfrac{1}{2}] + 1 = 0$$

$$\Rightarrow a_w^d = \tfrac{1}{4}\left[\frac{c - 2}{c}\right] = a^{d*}$$ ▲

In this setup the firm cannot affect its competitor's marginal cost (and hence its own delivered price) by relocating, given its rival's location. The only criterion determining its location decision is minimization of its transport cost, which is why the profit maximizing Nash equilibrium location of the firms coincide with the welfare maximizing location. We extend the cost function in (2.34) to the case where $c_i = (1 - d)q_i, \quad i = 1, 2$ ...(2.38)

where $d = 1 - a - b$. Let p_i^{dq*} denote the equilibrium price schedule for firm i and a^{dq*} and b^{dq*} represent the equilibrium locations for firms 1 and 2 respectively. With this cost function, the negative distance externality affects marginal cost. Solving by backward induction for the subgame perfect price and location equilibria, we obtain the results in Proposition 2.7 below.

Proposition 2.7

There exists an unique equilibrium delivered price schedule such that $p_1^{dq*}(x, a, b) = p_2^{dq*}(x, a, b) = \max\,[c|x - a| + (a + b), c|1 - b - x| + (a + b)]$. The symmetric Nash equilibrium locations are at the first and third quartiles of the market.

Proof: Part 1: Price equilibrium: Along the lines of proof of Proposition 2.1.

Part 2: Location equilibrium

The profit functions for firm 1 and 2 are

$$\pi_1^{dq*}(a, b) = \int_0^{\bar{x}} [p_1^{dq*} - c|x - a|]\, dx - c_1(q_1, d) \qquad \text{...(2.39)}$$

$$\pi_2^{dq*}(a, b) = \int_{\bar{x}}^{1} [p_2^{dq*} - c|1 - b - x|]\, dx - c_2(q_2, d) \qquad \ldots(2.40)$$

where $\bar{x}$ is the consumer location with equal delivered prices for both firms. That is,

$$p_1(x, a, b) = p_2(x, a, b)$$

$$\Rightarrow c(\bar{x} - a) + (a + b) = c(1 - b - \bar{x}) + (a + b)$$

$$\Rightarrow \bar{x} = \frac{1 + a - b}{2} \qquad \ldots(2.41)$$

Substituting p_1^{dq*}, p_2^{dq*} and (2.38) in (2.39) and (2.40) we get

$$\pi_1^{dq*}(a, b) = \int_{0}^{\bar{x}} [c(1 - b - x) + (a + b) - c|x - a|]\, dx - (a + b)q_1 \qquad \ldots(2.42)$$

$$\pi_2^{dq*}(a, b) = \int_{\bar{x}}^{1} [c(x - a) + (a + b) - c|1 - b - x|]\, dx - (a + b)q_2 \qquad \ldots(2.43)$$

Let us consider profit of firm 1. Using (2.18) in (2.42), we get

$$\pi_1^{dq*}(a, b) = \int_{0}^{\bar{x}} [c(1 - b - x) - c|x - a|]\, dx + (a + b) \int_{0}^{\bar{x}} x'dx' -$$

$$(a + b) \int_{0}^{\bar{x}} x'dx'$$

$$= \int_{0}^{\bar{x}} [c(1 - b - x) - c|x - a|]\, dx$$

The first-order condition implies

$$\frac{\partial \pi_1^{dq*}(a,b)}{\partial a} = -2ac + c\bar{x} + c[1 + a - b - 2\bar{x}]\, \frac{\partial \bar{x}}{\partial a} = 0$$

Substituting (2.14),

$$\Rightarrow -2ac + c[\frac{1+a-b}{2}] + \tfrac{1}{2}c[1 + a - b - 2(\frac{1+a-b}{2})] = 0$$

$$\Rightarrow -2ac + c[\frac{1+a-b}{2}] = 0$$

$\Rightarrow a^{dq^*} = \frac{1}{4}$. By symmetry, $b^{dq^*} = \frac{1}{4}$. ▲

Each firm has constant marginal costs, given firm locations a and b. From the discussion in section 2.3, we know that the symmetric profit maximizing locations would be at the quartiles.

However, the negative distance externality has welfare implications. Given each firm's location, the rival firm can relocate in a way which will raise the other firm's marginal cost and hence, its own price. Clearly in this situation, the welfare maximizing location would not coincide with the profit maximizing location. This result is summarized in the proposition below.

Proposition 2.8

When the distance between the two firms creates a negative externality that affects marginal cost of production. welfare maximizing locations lie outside the quartiles and do not coincide with the locations chosen by profit maximizing firms.

Proof: Let a_w^{dq} denote firm 1's welfare maximizing location. Using (2.38).

$$TC(W) = c_1(d, q_1) + c_2(d, q_2) + 2c \int_0^{1/2} |x-a| \, dx.$$

$$TC(W) = \sum_{i=1}^{2}[(a+b)q_i] + 2c \int_0^{1/2} |x-a| \, dx$$

The first-order condition implies

$$\frac{\partial TC(W)}{\partial a} = \sum_{i=1}^{2} [(a + b) \frac{\partial q_i}{\partial a}] + \sum_{i=1}^{2} q_i + 2c[2a - \tfrac{1}{2}] = 0 \qquad \ldots(2.44)$$

$$\text{We know } \frac{\partial q_1}{\partial a} = \frac{\partial q_2}{\partial a} \text{ and } q_1 + q_2 = 1 \qquad \ldots(2.45)$$

Using (2.45) in (2.44)

$$\Rightarrow 1 + 2c[2a - \tfrac{1}{2}] = 0$$

$$\Rightarrow a_w^{dq} = \tfrac{1}{4}[\frac{c - 1}{c}] < \tfrac{1}{4}$$

We have discussed three cases of externalities; externalities in output that affect marginal cost, fixed cost externalities in distance and finally an externality in distance which affects marginal cost. We conclude that as long as marginal costs are affected by a rival firm's decisions on location, the profit maximizing location does not minimize total production and transport costs and hence does not maximize welfare.

Section 2.5: Conclusions

This paper analyzes a duopoly model where the firms play a two-stage game in location and price. They have the ability to set discriminatory prices in the presence of inelastic demands and nonconstant marginal costs of production. The traditional assumption of zero or constant marginal costs of production gave the result that profit maximizing firms would always locate so that they minimized transport cost. It is shown that with nonconstant marginal costs of production, although social welfare will be maximized when they so locate, firms will not generally have an incentive to minimize transport cost alone, since a firm's profit will depend upon how its location will affect its rival's production cost as well as its own production and transportation cost. With nonlinear production costs, each firm would want to relocate in a manner which raises its rival's marginal cost and hence its own delivered price. The case of nonconstant marginal costs of production is then extended to a problem of output externality and the resulting pattern of location equilibria is analyzed. Finally, we conclude by considering a negative externality in distance between the firms and we prove that the resulting equilibrium in fact maximizes social welfare.

The motivation for a production externality becomes more meaningful in the case of two different industries, each comprised of say two firms. We can construct, in this setup, an asymmetric inter-industry externality, assuming that intra-industry externality is zero. This model would be a generalization of the present framework in two ways: In incorporating nonidentical products and in allowing asymmetric externalities. We know that a real externality creates a divergence between private optima and social optima. In the literature, taxation is one way of restoring the socially optimal equilibrium. In this context, it would be interesting to search for such a tax structure, which will induce the firms to choose locations that are socially optimal.

CHAPTER 3
SEQUENTIAL ENTRY AND DETERRENCE WITH SPATIAL PRICE DISCRIMINATION

Section 3.1: Introduction and Literature Review

It is well known that by undertaking an observable credible commitment an incumbent firm can induce rivals to respond less aggressively and thereby enhance its own profits. With spatial competition, a once-and-for-all location choice can constitute such a credible commitment. In the real world a sequence of firms enter a developing industry at different points in time. Yet, most of the literature assumes simultaneous entry. Clearly sequential entry is far more realistic than simultaneous location choice.

In this paper, we address the issue of sequential entry in a standard price discrimination model (Lederer and Hurter,1985). They discussed a two-stage spatial price discrimination model, with simultaneous entry in stage one, followed by price competition in the second stage. The standard analysis is extended to allow earlier entrants to choose locations noncooperatively so as to deter subsequent entry. We examine a two-stage game: in the first stage firms enter the industry sequentially and choose permanent locations along a bounded linear market. In the second stage, equilibrium prices are chosen simultaneously. An implicit assumption of sufficiently low menu costs makes simultaneous price choice a reasonable assumption. Firms are assumed to have perfect foresight regarding the behavior of subsequent competitors.

Sequential entry in a spatial model has been studied by Prescott and Visscher (1977). In their analysis, price is fixed, so that location is the single choice variable. They modify the standard Hotelling framework by assuming that firms locate sequentially in the market, with each firm having perfect foresight about the optimal actions of those firms entering later in the sequence. Kats (1986) generalized an example from Prescott and Visscher. He determined the n firm entry equilibrium with fixed prices. In this equilibrium, the first n-1 agents are spaced at their profit maximizing locations while the last player locates randomly. In a subsequent paper, Anderson (1987) extends the Prescott and Visscher analysis to a duopoly model by making price an endogenous variable. He overcomes the problem of nonexistence of price equilibria at certain locations by considering a price leadership model. The problem with the Hotelling (1929) model was that there was no price equilibrium for ceratin location choices. This point was first demonstrated analytically by d'Aspremont, Gabszewicz and Thisse (1979). Anderson (1977) demonstrated that in equilibrium the first firm locates at the midpoint of the market. The second firm then enters relatively close to one of the ends of the market and elects to act as a price leader.

When firms enter sequentially, earlier entrants can often profitably preempt subsequent entry by strategically erecting entry barriers. However, in the literature on spatial models, the analysis of strategic entry deterrence has been very limited. Gerlowski (1988) examines a collusive oligopoly model in a Hotelling framework. When entry is threatened, the colluding firms react by charging an entry deterring price, which is never less than the marginal cost of production. In the post-entry equilibrium, firms located nearer the entrant charge a lower price than firms farther away. Neven (1987) uses numerical simulation to study sequential entry in the Hotelling model and concludes that earlier entrants locate systematically around the center, but in a way that will induce later competitors not to locate between them. Further, he allows location to be an entry deterrent and concludes that deterrence strategies lead firms to locate at more regular intervals.

In this paper, we consider entry of an exogenously specified number of firms. In a duopoly, there is an asymmetric sequential location equilibrium, in direct contrast to existing results for this case. With three firms, the first two entrants locate symmetrically around the third firm. Thus in a duopoly, the earliest entrant earns a higher profit, while with three firms, the first two entrants earn equal profits which are higher than that of the third firm. That is, profits are nonincreasing in order of entry and as the number of firms increases from two to three, the first entrant loses some of its "first mover advantage" over the second firm. The analysis is extended to make the entry process endogenous. We consider a framework where locations are chosen strategically by the incumbent firm(s) to profitably deter entry. This becomes possible since, in the presence of a sunk cost, the profit of a potential entrant can be made negative for certain strategic locations of the incumbent firms. It is shown that in this process of endogenous sequential entry, there are ranges of sunk cost (depending on the number of incumbent firms) for which firms deter entry by locating at their unconstrained profit maximizing locations; there are ranges of sunk cost for which early entrants choose locations to create a strategic entry barrier. That is, they choose their profit maximizing location subject to the constraint of entry deterrence. We derive and discuss the properties of strategic sequential location and price equilibria.

The rest of the paper is organized as follows: In section 3.2, we present the model. Section 3.3 discusses exogenous sequential location choice and the price equilibrium for two and three firms respectively. It also compares the profit maximizing location equilibria with the corresponding welfare maxima. In section 3.4, we discuss endogenous sequential location choice, allowing entry deterring strategies. Finally, section 3.5 is a summary of the findings with suggestions for future extensions.

Section 3.2: The Model

In this model, identical consumers are uniformly distributed over a linear market of unit length. Each consumer has a perfectly inelastic demand for one unit of the homogenous good, with reservation price r.

The firms in the industry play a two-stage game. In stage one, firms enter the industry sequentially and choose a location in the market. Costs of relocation are assumed to be prohibitively high, so location choices are irreversible decisions. In the second stage, price competition takes place. Firms can charge discriminatory prices to consumers, based on the relative location of the consumer vis-a-vis all the firms. It is assumed that all entrants can compute the second stage price equilibrium as a function of where they locate. The appropriate equilibrium in the first stage is therefore a subgame perfect Nash equilibrium in location and price. Such an equilibrium can be solved by backward induction, first finding the price schedules resulting from any location pattern and then determining locations in reverse order of entry. All firms have perfect foresight regarding the behavior of potential entrants.

The firms have a constant (normalized to zero) marginal cost of production and a positive sunk cost of entry, F. Transport cost is linear in the distance transported, where c is the cost per unit distance. If a consumer faces an identical price schedule from two firms. he buys from the firm which is located closer to him. This is an ϵ-equilibrium. If a consumer buys from the firm located further away, the nearer firm can undercut its delivered price by ϵ. In the model, welfare maximizing locations are those which minimize total transport cost.

Let l_1, l_2 and l_3 denote the locations of the entrants in their order of entry, where l_1 is the distance from from the left market boundary while l_2 and l_3 are measured from the right boundary of the market. Let $p_i(x, l_i, l_{(-i)})$ denote the delivered price schedule offered by firm i to a consumer located at x, measured from the left market boundary; l_i is the location of the i^{th} firm; $l_{(-i)}$ is the location vector of the other firms in the market. First consider the price schedules. A price schedule $(p_i^*, p_{(-i)}^*)$ is a Nash equilibrium given locations $(l_i, l_{(-i)})$, if and only if $\pi_i^*(p_i^*(x, l_i, l_{(-i)}), p_{(-i)}^*(x, l_i, l_{(-i)})) \geq \pi_i (p_i(x, l_i, l_{(-i))}, p_{(-i)}^*(x. l_i, l_{(-i)}))$ for all i. Profit can be expressed in terms of firm locations. Next, we consider the equilibrium locations of the firms. A set of locations

$(l_i^*, l_{(-i)}^*)$ would form a Stackelberg equilibrium if and only if

$$l_i^* = \arg\max_{l_i} \pi_i (l_i | l^*_{(-i)})$$

where $l^*_{i+1} = \arg\max_{l_{i+1}} \pi_{i+1} (l_{(i+1)} | l^*_{(-(i+1))})$, for all i.

Section 3.3: Exogenous Sequential Entry

In this section, we assume that the number of firms in the industry is fixed. For instance, sunk costs are already paid before stage one of the game. However no profits are realized until entry is completed. The presence of a sunk cost ensures that at most a certain fixed number of firms will enter the industry. From the equilibrium solution, each firm can then determine the actual number of entrants.

In the following proposition we first look at the welfare maximizing sequential locations. The welfare maximizing locations would be those that minimize total transport cost. The proposition below summarizes the results for n = 2 and n = 3.

Proposition 3.1

With n = 2, the welfare maximizing locations are at the first and third quartiles of the market. With n = 3, the welfare maximizing locations are such that $l_1^w = l_2^w = 1/6$ and $l_3^w = 1/2$.

Proof: n = 2: Proof is along the lines of proof for n = 3, below.

n = 3:

$$\text{Total Cost (TC)} = \sum_{i=1}^{3} TC_i$$

$$= c[\int_{0}^{\bar{x}_{13}} |x - l_1| \, dx + \int_{x_{23}}^{1} |1 - l_2 - x| \, dx + \int_{x_{13}}^{\bar{x}_{23}} |1 - l_3 - x|] \, dx$$

where $\bar{x}_{13} = \dfrac{1 + l_1 - l_3}{2}$

$$\bar{x}_{23} = 1 - \frac{(l_2 + l_3)}{2}$$

$$\bar{x}_{12} = \frac{1 + l_1 - l_2}{2}$$

Taking first-order conditions we get

$$\frac{\partial TC}{\partial l_3} = 0 \Rightarrow l_3 = \frac{1 - l_1 + l_2}{2} \qquad \ldots(3.1)$$

$$\frac{\partial TC}{\partial l_2} = 0 \text{ and using (3.1)} \Rightarrow l_2 = \frac{1 - l_1}{5} \qquad \ldots(3.2)$$

$\dfrac{\partial TC}{\partial l_1} = 0$ and substituting (3.1) and (3.2)

$\Rightarrow l_1^w = 1/6$. Solving (3.1) and (3.2) we get

$l_2^w = 1/6$

$l_3^w = 1/2$.

It is straightforward to check that second-order conditions are satisfied. ▲

In effect, this result provides a benchmark to facilitate interpretation of the derived inefficiencies when we consider the equilibrium locations with n = 2 (Proposition 3.3) and n = 3 (Proposition 3.5). We will show that the sequential location framework yields equilibria which do not maximize welfare.

The sequential location equilibria for $n = 2$ and $n = 3$ are derived as are the corresponding subgame perfect Nash equilibrium price schedules. The profit maximizing location equilibria are then compared to the welfare maximizing location equilibria.

3.3.1: Sequential Entry Without Deterrence for n = 2

Consider an industry where there are two firms. For subgame perfect equilibria we use backward induction to derive stage two price schedules first. The equilibrium price schedules, in essence, have each firm charging the maximum of both firms' transport costs.

Proposition 3.2

There exists a unique Nash equilibrium price schedule such that

$$p_1^*(x,\ l_1,\ l_2) = p_2^*(x,\ l_1,\ {}_2) = \max\ [c|x - l_1|,\ c|1 - l_2 - x|]$$

Proof: (Due to Lederer and Hurter, 1985)

For any fixed locations l_1, l_2 and price schedule $p_2(x, l_1, l_2)$ for firm 2, an optimal price schedule for firm 1 is

$$p_1(x,\ l_1,\ l_2) = \max\ [p_2(x,\ l_1,\ l_2) - \epsilon,\ c|x - l_1|] \qquad \ldots(3.3)$$

where $\epsilon > 0$ and is arbitrarily small. Furthermore, (3.3) assumes that firm 1 will not price below it's marginal cost.

Firm 2 knows that firm 1 will choose (3.3) and thus firm 2 will select

$$p_2^*(x,\ l_1,\ l_2) = \max\ [c|x - l_1| - \epsilon',\ c|1 - l_2 - x|] \qquad \ldots(3.4)$$

Firm 1's optimal response will be

$$p_1^*(x,\ l_1,\ l_2) = \max\ [c|1 - l_2 - x| - \epsilon,\ c|x - l_1|] \qquad \ldots(3.5)$$

Firms 1 and 2 seek to make ϵ, ϵ' arbitrarily small. In the limit, as ϵ, $\epsilon' \rightarrow 0$,

$$\Rightarrow p_1^*(x,\ l_1,\ l_2) = p_2^*(x,\ l_1,\ l_2) = \max\ [c|x - l_1|,\ c|1 - l_2 - x|] \qquad ▲$$

Under Bertrand price competition, given x, l_1 and l_2, equilibrium delivered price schedules of both firms must be equal. Given the assumption that a consumer facing equal delivered prices will buy from the firm located closer to him, it follows that in equilibrium the maximum price that a firm can charge a consumer is equal to the transport cost of the firm located closest to that consumer.

We now turn to stage one of the game. Firm 1 enters first and chooses its profit maximizing location knowing that firm 2 will subsequently enter the market. The sequential equilibrium is summarized in Proposition 3.3.

Proposition 3.3

The sequential duopoly locations are $l_1^* = 2/5$, $l_2^* = 1/5$.

Proof: Let firm 1 be the leader in the first stage sequential location choice, choosing location at l_1. Firm 2 follows, choosing to locate at l_2, given l_1. With foresight, firm 1 incorporates firm 2's reaction function in its profit function, to derive its profit-maximizing Stackelberg location.

Let $\bar{x}$: consumer location where the two firms' transport costs are equal.

$$\Rightarrow \bar{x} = \frac{(1 + l_1 - l_2)}{2} \qquad \ldots(3.6)$$

To derive firm 2's reaction function:

$$\pi_2(l_1, l_2) = \int_{\bar{x}}^{1} [p_2^*(x, l_1, l_2) - c\,|1 - l_2 - x|]\, dx$$

Substituting $p_2^*(x, l_1, l_2)$ from Proposition 3.2,

$$= \int_{\bar{x}}^{1-l_2} [c(x - l_1) - c(1 - l_2 - x)]\ dx + \int_{1-l_2}^{1} [c(x - l_1) - c(x - 1 + l_2)\ dx$$

Taking first-order condition and simplyfying, we get

$$\frac{\partial \pi_2}{\partial l_2} = \frac{(1 - 3l_2 - l_1)}{2} = 0$$

$$\Rightarrow l_2 = (1 - l_1)/3 \qquad \ldots(3.7)$$

Next we consider firm 1's location choice.

$$\pi_1(l_1,\ l_2(l_1)) = \int_0^{\bar{x}} [p_1^*(x,\ l_1,\ l_2) - c\,|x - l_1|]\ dx$$

Substituting for p_1^* from Proposition 3.2,

$$\pi_1(l_1) = \int_0^{l_1} [c(1 - l_2 - x) - c\ (l_1 - x)]\ dx + \int_{l_1}^{\bar{x}} [c(1 - l_2 - x) - c(x - l_1)]dx$$

Substituting (3.6) and (3.7) in the above expression, taking first-order condition and simplyfying, we get

$$\frac{\partial \pi_1(l_1)}{\partial l_1} = -\ 20l_1 + 8 = 0$$

$$\Rightarrow l_1^* = 2/5$$

From (3.7) we get

$l_2^* = 1/5$.

It is straightforward to check that second-order conditions are satisfied.▲

Interestingly enough an asymmetric equilibrium arises in a duopoly model when firms choose their locations in sequence. This is in contrast to the simultaneous entry game in which the duopolists locate symmetrically at the quartiles of the market (Lederer and Hurter, 1985).

The asymmetry here arises from the "first mover advantage" enjoyed by the early entrant, which yields higher profits to him. Anderson (1987) also obtained an asymmetric equilibrium in a duopoly model, with sequential entry and price leadership, where there is mill pricing instead of price discrimination. As shown here, with discriminatory pricing, sequential entry alone (and simultaneous price choice) is sufficient to generate an asymmetric equilibrium in terms of location and profits. A second notable property of the location equilibrium is that the first firm does not locate at the center of its market. Thus the "market median property" which is typically observed in spatial models, does not arise here.

As the number of firms is increased to three, the sequential location equilibria tend to look more symmetric. The market with three firms is discussed in subsection 3.3.2 below.

3.3.2: Sequential Entry Without Deterrence for n = 3

In this section we consider a market which supports three firms which enter sequentially. As before, in stage two, they simultaneously choose delivered price schedules. The stage two Nash equilibrium delivered price schedule is summarized in Proposition 3.4 below:

Proposition 3.4

There exists a unique Nash equilibrium price schedule such that $p_i^*(x, l_i, l_j, l_k) = \max\ [c|x - l_i|, \min(c|x - l_j|, c|x - l_k|)]$ where $i \neq j \neq k = 1, 2, 3$.

Proof: Similar to proof of Proposition 3.2. See figure 3.1.

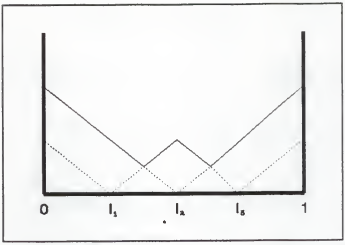

Figure 3.1

Let l_1, l_2, l_3 be the first stage sequential locations chosen by the three firms in this setting. The solid lines in figure 3.1, represent the equilibrium price schedule. The dotted lines represent each firm's transport cost. It is clear that firm i will choose a delivered price schedule which will be the maximum of: (1) its own delivered marginal cost and (2) the minimum of the delivered marginal cost of its competitors. That is, its price schedule will be the maximum of its own delivered marginal cost and the delivered marginal cost of its nearest rival.

We now consider the sequential location equilibrium with three firms. The result is summarized in Proposition 3.5 below:

Proposition 3.5

The sequential equilibrium locations are: $l_1^* = l_2^* = 0.275255$, $l_3^* = 0.5$.

Structure of the proof.

Define l_1^* as firm 1's location, corresponding to which $\exists\ l_2^*$ such that $\pi_3^1 = \pi_3^2 = \pi_3^3$ (where the subscript denotes the third firm by order of entry and the superscript denotes the first, middle and third slot). That is, firm 3 makes equal profits by locating in either one of the three available slots.

Thus firm 1 can choose either Case 1: $l_1 \leq l_1^*$ or

Case 2: $l_1 > l_1^*$ with foresight regarding firm 2's location in each case.

Consider Case 1: $l_1 \leq l_1^*$

$\Rightarrow \pi_3^1 \leq \pi_3^2$

Clearly, $\pi_3^1 < \pi_3^2$ can be ruled out, since firm 1 would be better off moving to the right, increasing π_3^1 and decreasing π_3^2 till firm 3 is indifferent between slots 1 and 2.

Therefore, Case 1 reduces to $l_1 = l_1^*$.

If $l_1 = l_1^* \Rightarrow l_2$ is such that $\pi_3^1 = \pi_3^3 \lesseqgtr \pi_3^2$

To prove: l_2^* is such that $\pi_3^1 = \pi_3^3 = \pi_3^2$

Suppose $\pi_3^1 = \pi_3^3 \neq \pi_3^2$

Let $\pi_3^1 = \pi_3^3 > \pi_3^2$.

$\Rightarrow$ firm 3 is strictly better off by entering either to firm 1's left or to firm 2's right; that is, in their respective monopoly regions. Clearly firms 1 and 2 are not behaving optimally in this case, since they could increase their own profits by making firm 3 indifferent to locating in the center.

Let $\pi_3^1 = \pi_3^3 < \pi_3^2$. Clearly firms 1 and 2 could relocate to make firm 3 indifferent to locating in the middle slot.

$\Rightarrow l_1^*$ and l_2^* are such that $\pi_3^1 = \pi_3^2 = \pi_3^3$

Therefore, if $l_1 = l_1^*$ then $l_2 = l_2^*$ such that $\pi_3^1 = \pi_3^2 = \pi_3^3$.,

Case 2: $l_1 > l_1^*$

Then firm 2 will choose $l_2 = l_1 - \epsilon$ where $\epsilon \to 0$

such that $\pi^3{}_3 = \pi^1{}_3 - \Delta > \pi^2{}_3$ where $\Delta \to 0$.

That is, firm 2 locates such that firm 3 is better off by locating to firm 1's left; i.e. in its monopoly region. Finally combining Case 1 and Case 2 we will show that, in equilibrium, $l_1 = l_1^*$, $l_2 = l_2^* = l_1^*$, $l_3^* = 0.5$.

Proof: Case 1: $l_1 = l_1^*$

Step 1: To derive l_1^*, l_2^*.

l_1^* is such that $\pi^1{}_3 = \pi^2{}_3$

$\Rightarrow (1/3)\ l_1^2\ c = (1/8)\ (1 - l_1 - l_2)^2\ c$

Simplyfying, we get

$l_1^* = 0.3797958\ (1 - l_2)$ ...(3.8)

l_2^* is such that $\pi^2{}_3 = \pi^3{}_3$

$\Rightarrow l_2^* = 0.3797958\ (1 - l_1)$ ...(3.9)

Solving (3.8) and (3.9) for l_1^* and l_2^*, we get

$l_1^* = l_2^* = 0.275255$. ...(3.10)

Step 2: To derive l_3 in terms of l_1 and l_2.

When l_1 and l_2 are such that $\pi^1{}_3 = \pi^2{}_3 = \pi^3{}_3$, clearly firm 3 is indifferent between the three slots. We assume that, in that case, firm 3 locates in the middle slot. It is straightforward to argue that firm 3 locates in the center of its market; i.e. it is equidistant from firms 1 and 2.

$\Rightarrow l_3 = \frac{1}{2}\{1 - l_1 + l_2\}$ ...(3.11)

Step 3: To derive $\pi_1(l_1^*, l_2^*, l_3^*) = \frac{1}{2}\{1 - l_1 + l_2\}$

$$\pi_1 = \int_0^{\bar{x}_{13}} [c(1 - l_3 - x) - c|x - l_1|]\, dx$$

$$= \int_0^{l_1} [c(1 - l_3 - x) - c(l_1 - x)]\, dx + \int_{l_1}^{\bar{x}_{13}} [c(1 - l_1 - x) - c(x - l_1)]\, dx$$

Simplyfying, we get

$\pi_1 = c[0.0248408 + 0.2620205 l_1 - 0.2860613 l_1^2]$

From first and second-order conditions we get,

$$\frac{\partial \pi_1}{\partial l_1} = 0 \Rightarrow l_1 = 0.4579796$$

$$\frac{\partial^2 \pi_1}{\partial l_1^2} < 0$$

In figure 3.2, the X axis represents firm 1's location. The Y axis represents firm 1's profit, $\pi_1(l_1)$.

From figure 3.2, $\forall\ l_1 < 0.45$, $\frac{\partial \pi_1}{\partial l_1} > 0$

From (3.10) we have $l_1^* = l_2^* = 0.275255$

From Step 2 we know $l_3^* = 0.5$

Hence $\pi_1(l_1^*, l_2^*, l_3^*) = 0.0744897c$ ...(3.12)

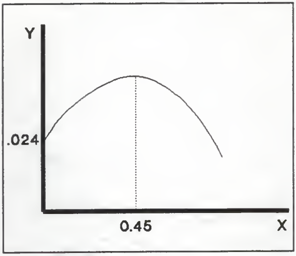

Figure 3.2

Case 2: $l_1 > l_1^*$

We argued above that this implies $l_2 = l_1 - \epsilon$,

subject to $\pi^1_3 > \pi^3_3 > \pi^2_3$.

Step 1: To derive $l_3(l_1, l_2)$

$$\pi_3 = \int_0^{1-l_3} [c(l_1 - x) - c(1 - l_3 - x)]\, dx$$

$$+ \int_{1-l_3}^{\bar{x}_{13}} [c(l_1 - x) - c(x - 1 + l_3)]dx$$

where $\bar{x}_{13} = \dfrac{1 + l_1 - l_3}{2}$

Simplyfying and taking first-order condition, we get

$$\frac{\partial \pi_3}{\partial l_3} = 0 \Rightarrow l_3 = 1 - \frac{l_1}{3} \qquad \ldots(3.13)$$

Step 2: To derive $\pi^c{}_1(l_1 > l_1^*,\ l_2 = l_1 - \epsilon,\ l_3)$ where the superscript c denotes firm 1's profit in the center.

$$\pi_1{}^c = \int_{\bar{x}_{13}}^{\bar{x}_{23}} [c(x - 1 + l_3) - c(l_1 - x)]\, dx$$

$$+ \int_{x_{23}}^{l_1} [c(1-l_2 - x) - c(l_1 - x)]\, dx$$

$$+ \int_{l_1}^{\bar{x}_{12}} [c(1-l_2 - x) - c(x - l_1)]\, dx$$

where $\bar{x}_{13} = \dfrac{1 + l_1 - l_3}{2}$

$\bar{x}_{23} = 1 - \dfrac{(l_2 + l_3)}{2}$

$\bar{x}_{12} = \dfrac{1 + l_1 - l_2}{2}$

Substituting and simplyfying, we get

$\pi^c{}_1 = \frac{1}{2}\, c(1 - l_1 - l_2)\,(-1 + l_1 + l_3)$

Further, substituting $l_2 = l_1 - \epsilon$ and (3.13), we get

$\pi^c{}_1 = (1/3)cl_1\,[1 - 2l_1 + \epsilon]$ ...(3.14)

From Case 1, equation (3.12), we know

$\pi_1(l_1{}^*) = 0.0744897c$

Step 4: If $\pi_1\,(l_1{}^*) > \pi^c{}_1\,(l_1{}^* + \epsilon) \quad \forall\ \epsilon > 0$

$\Rightarrow l_1{}^*$ is firm 1's optimal choice.

From (3.14), $\pi_1{}^c(l_1{}^* + \epsilon) = c[0.0412 + 0.058\epsilon - 0.33\epsilon^2]$

Therefore, $\pi_1(l_1{}^*) - \pi_1{}^c(l_1{}^* + \epsilon)$

$= c[0.0744897 - 0.0412 - 0.058\epsilon + 0.33\epsilon^2]$

$= c[0.033 - 0.058\epsilon + 0.33\epsilon^2]$

Further $\frac{\partial(\pi_1 - \pi_1^c)}{\partial \epsilon} = -\,0.058 + 0.66\epsilon = 0 \;\Rightarrow\; \epsilon = 0.088$

$$\frac{\partial^2 (\pi_1 - \pi_1^c)}{\partial \epsilon^2} > 0$$

In figure 3.3, the X axis represents ϵ and the Y axis represents $\pi_1 - \pi_1^c$. From figure 3.3, it is evident that

$\pi_1 - \pi_1^c > 0 \;\; \forall\, \epsilon > 0 \;\Rightarrow\; l_1^*$ is firm 1's optimal choice.

Hence the equilibrium locations are $l_1^* = l_2^* = 0.275255$, $l_3^* = 0.5$.

The equilibrium profits are $\pi_1^* = \pi_2^* = 0.0744898c$, $\pi_3^* = 0.025255c$ ▲

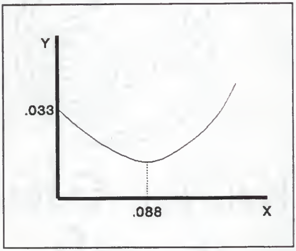

Figure 3.3

There are several interesting features of this location equilibrium. First, the initial two entrants locate symmetrically while the last entrant locates at the center of the market. Firms 1 and 2 earn the same profit while the last entrant earns less. Compared to the duopoly location equilibrium (Proposition 3.3), the first entrant shares some of its "first mover advantage" with the second entrant. Intuitively we would expect payoffs to be positively correlated to order of entry, which is true in the case of duopoly. But here, firms 1 and 2 have equal payoffs, so profits are only weakly correlated. Second, early entrants again do not locate at the median locations of their respective market areas. Neither firm locates at the Pareto efficient location, in contrast to the case of mill pricing where the first n-1 firms locate efficiently (Kats, 1986). Intuitively, in the case of duopoly, both firms knew that they would be the only ones in the market. Clearly firm 1 uses its first mover advantage to move to the right of 1/4 (the simultaneous entry location equilibrium) and firm 2 is consequently pushed to the right of 1/4 on its side of the market.

With three firms, however, the situation is significantly different. Firm 1 anticipates the reaction of two more entrants. Thus it is constrained to choose l_1 so that no later entrant will find it profitable to "jump in" between 0 and l_1. By symmetry, firm 2 does exactly the same thing. Given l_1, firm 2 chooses l_2 such that it is unprofitable for firm 3 to locate between $(1 - l_2)$ and 1. In other words, firm 1 chooses l_1 to constrain l_2 and l_3 to be to the right. Given l_1, firm 2 chooses l_2 to constrain l_3 to be to the left. In effect both firm 1 and firm 2 do exactly the same thing vis-a-vis firm 3--constrain it to choose a location between them. (Contrast this result with Neven (1987) where the firms use mill pricing. Recall that there the earlier entrants located around the center without allowing later entrants to locate between them.) Obviously firm 1 and firm 2 are better off maximizing the market area over which they have a monopoly. Firm 1 cannot earn higher profits than firm 2 because then firm 2 could locate in a way that made it profitable for firm 3 to jump in between $[0, l_1]$--thus pushing firm 1 in the center and making it worse off. It is clear that firm 2 implicitly uses firm 3 as a lever in detracting from the "first mover advantage" that would otherwise be enjoyed by firm 1. This leads to the symmetric locations (and equal payoffs) of the first two entrants.

As for equilibrium prices and profits, it is clear that a greater number of firms leads to more intense price competition. This leads to lower prices and lower profits with the firm in the center being worst off. Also notice that the "middle" firm has to offer two sets of prices: to its left, the delivered price of firm 1; to its right, the delivered price of firm 2. Finally, comparing Proposition 3.1 with Propositions 3.3 and 3.5, the profit maximizing sequential locations, in general, do not maximize welfare. However, with $n = 3$, the last entrant locates optimally in the center of the market.

Section 3.4: Sequential Entry With Deterrence

In this section we extend the sequential entry paradigm with perfect foresight, to the case of endogenous sequential entry. Earlier entrants use location choice strategically to preempt further entry. Specifically, each firm chooses a location to maximize profits, recognizing that its location choice will affect the potential profits that rivals can earn, and thus will affect the number of entrants. After observing location choices of earlier entrants, firms pay sunk costs if they enter. An equilibrium set of locations and prices requires that an incumbent firm cannot, by any alternative choice of location and/or price, increase its profits and that potential entrants cannot profitably enter. In this context, once-and-for-all location choice is a crucial assumption. Potential entrants calculate post entry profit with the information that existing firms have a credible threat in their strategic choice of location and will not accomodate a new firm that does enter. As we will show, there is a range of sunk cost (varying with the number of incumbents) where entry deterrence is simultaneously a feasible and profitable strategy. It is important to point out that entry deterring strategies are noncooperative. That is firms do not collude to keep a potential entrant out of the market. With foresight, all firms recognize when strategic entry deterrence is feasible and sequential entry leads to a noncooperative strategic entry deterring location equilibrium.

Consider initially an industry which is characterized by a sunk cost that is too high to make entry profitable for any firm. In the next six cases discussed below (Lemma 3.1 - 3.6), we progressively lower sunk cost to analyze its effect on sequential entry in a scenario where firms deter entry if it is feasible and profitable to do so. [See figure 3.6]

Lemma 3.1

If $F \geq r - 0.25c$, the market cannot sustain any firms.

Proof: Consider the monopoly profit in this market. Clearly the monopolist will charge a price equal to reservation price and locate in the middle of the market, where its transport cost is minimized.

Let π_m = monopoly profit.

$$\therefore \pi_m = \int_0^{l_1} [r - c(l_1 - x)]\, dx + \int_{l_1}^{1} [r - c(x - l_1)]\, dx$$

Substituting $l_1 = 0.5$ and simplyfying, we get

$\pi_m = r - 0.25\ c$.

Clearly, if $F > r - 0.25\ c$, entry is unprofitable. ▲

Lemma 3.2

If $0.083c \leq F < r - 0.25c$, the industry is in equilibrium with a monopoly, which locates at $l_1^* = 0.5$.

If sunk cost is greater than the maximum profit that a potential entrant could earn, it would not enter. In this case the maximum profit that an entrant can earn when $l_1^* = 0.5$, is 0.083c. Hence the monopolist happens to deter entry by locating at its unconstrained location. Notice the entry deterring aspect here. For instance if the first entrant were to accomodate a rival, it could do so by locating at 0.4, and then entry is profitable for a second firm at $l_2 = 0.2$ as long as $0.083c \leq F < 0.12c$. However its monopoly profit maximization precludes profitable entry.

Proof: What is the maximum profit that an entrant can earn given that the incumbent firm is a monopoly with $l_1^* = 0.5$ and $p_1^* = r$? Let l_2 denote entrant's location (measured from the right side of the market).

$$\therefore\ \pi_2(l_2) = \int_{\bar{x}}^{1} [p^* - c|1 - l_2 - x|]\ dx$$

where $p^* = c(x - l_1)$ using Proposition 3.1.

$$\bar{x} = \frac{1 + l_1 - l_2}{2} \qquad \text{using (3.6)}$$

Solving for optimal l_2,

$$\frac{\partial \pi_2(l_2)}{\partial l_2} = 0 \Rightarrow l_2 = \frac{1 - l_1}{3} = 1/6.$$

Therefore $\pi_2(l_1 = 1/2,\ l_2 = 1/6) = 0.083c$. This is the maximum profit that a potential entrant can earn. Therefore if $r - 0.25c > F \geq 0.083c$, firm 1 is deterring entry of a potential entrant by locating at its unconstrained profit-maximizing location. [We assume that zero profit does not attract entry.] ▲

Lemma 3.3

If $0.053c \leq F < 0.083c$, the industry is in equilibrium with a duopoly with $l_1^* = 0.4$, $l_2^* = 0.2$ and both firms are choosing their unconstrained entry deterring locations.

For an industry with sunk costs in this range, the first entrant, with foresight, recognizes that a monopoly cannot be sustained. It also recognizes that its unconstrained location choice keeps a third firm out of the market. Its profit maximizing location is at $l_1^* = 0.4$, which means it moves away from the center, and the second entrant locates at $l_2^* = 0.2$. These locations constitute the unconstrained equilibrium with $n = 2$, discussed in section 3.3.1. If a third firm moved into one of the three "slots" it could earn a maximum profit equal to 0.053c, by locating to firm 1's left. Hence if F is higher than that, the first two firms need not be concerned about a third rival.

Proof: From Lemma 3.2, the maximum profit that an entrant can make if $l_1 = 0.5$, is 0.083c. Clearly, if $F < 0.083c$, monopoly cannot be sustained. With foresight, the first entrant recognizes that there will be subsequent entry in the market. From Proposition 3.2, the unconstrained duopoly equilibrium locations are $l_1^* = 2/5$, $l_2^* = 1/5$. If a third firm enters, it will locate in one of three possible slots. [See page 45.]

It is easy to recognize that $\pi_3^1 = \max\ [\pi_3^1,\ \pi_3^2,\ \pi_3^3]$

Therefore if $F > \pi_3^1$, a third firm will not enter.

$$\text{Now, } \pi_3^1 = \int_0^{1-l_3} [c(l_1 - x) - c(1 - l_3 - x)]\ dx$$

$$+ \int_{1-l_3}^{\bar{x}} [c(l_1 - x) - c(x - 1 + l_1)]\ dx$$

where $l_1 = 2/5$, $\bar{x} = \dfrac{1 + l_1 - l_3}{2}$

$l_3^* = 1 - l_1/3$ from (3.13).

Substituting and simplyfying, we get

$\pi_3^1 = 0.053c$.

[It can easily be shown that $\pi_3^2 = 0.02c$, $\pi_3^3 = 0.013c$.]

Hence if $F \geq 0.053c$, the duopolists will deter entry by choosing their unconstrained locations $l_1^* = 2/5$, $l_2^* = 1/5$. ▲

Clearly if sunk cost is less than 0.053c, it is potentially profitable for a third firm to enter if the first two entrants located as in a duopoly. However, as the next cases illustrate, it is now feasible for the first two entrants to pre-empt entry of a third firm by strategic choice of their locations.

Lemma 3.4

If $0.028c \leq F < 0.053c$, there is a duopoly equilibrium where firm 1 locates strategically at $l_1^* = \sqrt{(3F/c)}$. Firm 2 locates at its unconstrained best response, at $l_2 = (1-l_1)/3$.

Proof: Given an unconstrained duopoly (Lemma 3.3) where $l_1 = 2/5$, $l_2 = 1/5$ as a starting point, clearly if π_3^i denotes firm 3's profits in slot i, it is easy to show that $\pi^1{}_3 > \pi^2{}_3 > \pi^3{}_3$ [refer to Lemma 3.3]. That is, the slot to the left of firm 1 is the most profitable location. This implies that to deter entry, firm 1 has to move to the left until $\pi_3{}^1 \leq F \Rightarrow l_1 \leq \surd(3F/c)$. In terms of $\pi_3{}^i$, this means that $\pi_3{}^1 \leq \pi_3{}^2 > \pi_3{}^3$.

To derive firm 1's optimal location, $l_1{}^*$:

From firm 1's profit function,

$\pi_1 = c[1 - l_1 - l_2][\frac{1}{4} + \frac{3}{4} l_1 - \frac{1}{4} l_2]$

Substituting firm 2's optimal response, $l_2 = (1-l_1)/3$ from (3.7)

$\pi_1 = c[1/9 + (4/9)l_1 - (5/9)l_1{}^2]$

$$\Rightarrow \frac{\partial \pi_1}{\partial l_1} = 0 \Rightarrow \quad l_1 = 2/5$$

$$\frac{\partial^2 \pi_1}{\partial l_1{}^2} < 0$$

In figure 3.4, the X axis represents l_1 and the Y axis represents $\pi_1(l_1)$. From figure 3.4, firm 1 will locate as close as possible to its profit maximizing location $\Rightarrow l_1{}^* = \surd(3F/c)$. Also, $\Rightarrow \pi_3{}^1 = \pi_3{}^2 > \pi_3{}^3$.

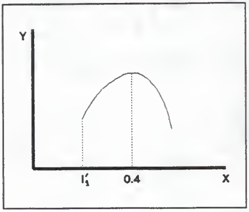

Figure 3.4

Therefore, firm 2 locates according to its best response since entry deterrence is not a binding constraint on its location choice.

Finally, $\pi_3^1 = \pi_3^2 \Rightarrow l_1' = 0.289897951$

$\Rightarrow \surd(3F/c) \geq 0.289897951$

$\Rightarrow F \geq 0.028c.$

Hence $\forall\ F \geq 0.028c,\ \ l_1^* = \surd(3F/c),\ l_2^* = (1-l_1)/3.$ ▲

Here the Stackelberg location equilibrium involves constrained location choice by firm 1, while firm 2 chooses its unconstrained location. In other words, firm 1 is deterring entry of the potential third entrant, while firm 2 is locating according to its best response to firm 1's location. Intuitively, it is most profitable for a third firm to enter to firm 1's left. To deter entry therefore, firm 1 has to move to its left (compared to the unconstrained duopoly equilibrium). Moreover, given this range of sunk cost, firm 2 does not face the threat of entry on either side if it chooses its best response location. In a sense therefore, entry deterrence cannot be forced onto subsequent entrants. In this case, the incumbent must pay the cost of entry deterrence itself.

Lemma 3.5

If $0.02525c \leq F < 0.028c$, both firms 1 and 2 locate to strategically deter entry with $l_1^* = (3F/c)^{½}$, $l_2^* = 1 - l_1 - (8F/c)^{½}$.

Proof: From the proof of Lemma 3.4, we know that firm 1 locates such that $\pi_3^1 = \pi_3^2$.

Recall that $\pi_3^1 = \pi_3^2 \Rightarrow l' = 0.289897951.$

$\Rightarrow$ if $l_1 \leq \surd(3F/c) < l_1' \Rightarrow \pi_3^1 < \pi_3^2 > \pi_3^3.$ ...(3.15)

This implies that entry deterrence is now a binding constraint on firm 2.

Firm 2 will choose l_2, subject to $\pi_3^2 \leq F$ ...(3.16)

and $\pi_3^3 \leq F$ ...(3.17)

(3.16) implies $(1/8)(1 - l_1 - l_2)^2 c \leq F \Rightarrow l_2 \geq 1 - l_1 - \sqrt{(8F/c)}$

(3.17) implies $(1/3) l_2^2\, c \leq F \Rightarrow l_2 \leq \sqrt{(3F/c)}$.

That is, from (3.16) and (3.17) we know

$1 - l_1 - \sqrt{(8F/c)} \leq l_2 \leq \sqrt{(3F/c)}$. ...(3.18)

From (3.15) we know $F < 0.028c$.

What is firm 2's optimal choice when $l_1 < l_1'$?

$$\pi_2(l_1, l_2) = \int_{x}^{1} [P_2^*(x, l_1, l_2) - c\,|1 - l_2 - x|]\, dx$$

Simplyfying, we get

$\pi_2 = c(1 - l_1 - l_2)(¼ - ¼l_1 + ¾l_2)$

Taking first and second-order conditions,

$$\frac{\partial \pi_2}{\partial l_2} = 0 \Rightarrow l_2 = \frac{1 - l_1}{3}$$

$$\frac{\partial^2 \pi_2}{\partial l_2^2} = -3/2$$

In figure 3.5, the X axis represents l_2 and the Y axis represents $\pi_2(l_2)$. Clearly, from figure 3.5, when $l_1 < l_1'$ and $l_2 \in [1 - l_1 - \sqrt{(8F/c)}, \sqrt{(3F/c)}]$, optimal l_2 is as close as possible to its unconstrained optimum.

$\Rightarrow l_2^* = 1 - l_1 - \sqrt{(8F/c)}$. Recall that $l_1^* = \sqrt{(3F/c)}$.

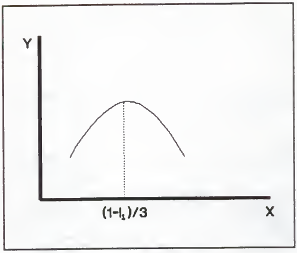

Figure 3.5

Finally, to derive the lower bound on F, which will support this equilibrium.

Definition: F_{min} is the value of sunk cost below which strategic entry deterrence is not feasible.

Clearly neither duopolist can deter entry if it is profitable for a third firm to enter, given that l_1, l_2 are such that $\pi_3^1 = \pi_3^2 = \pi_3^3$. It is easy to show that from the above constraint, $F_{min} = 0.025255c$. ▲

As sunk cost gets smaller, strategic entry deterrence by both firms becomes the profitable strategy. That is, while firm 1's entry deterring location pushes it to the left, entry is now attractive in the middle slot for the lower range of sunk cost, if firm 2 continues to choose its unconstrained best response. So, firm 2 now has to deter entry from the middle slot. Hence for this range of sunk cost, there is a constrained duopoly equilibrium, with both firms locating subject to the constraint of entry deterrence.

Lemma 3.6

If $F < F_{min}$ there exists equilibrium with $l_1^* = l_2^* = 0.275255$, $l_3^* = 0.5$.

Proof: From the definition of F_{min}, entry deterrence is not feasible for $F < F_{min}$. The first two entrants anticipate $n = 3$, and the unconstrained equilibrium locations are as proved in Proposition 3.5. ▲

There exists an equilibrium with $l_1^* = l_2^* = 0.27525$, $l_3^* = 0.5$. This is the unconstrained equilibrium with $n = 3$ discussed in section 3.3.2. $F_{min} = 0.02525c$. This is the lower bound on the range of sunk cost in which entry deterrence is feasible. If $F < F_{min}$ the first two entrants cannot deter entry of the third firm. With foresight the best they can do is accomodate him. (Note that we have implicitly allowed a lower bound on F which keeps a fourth entrant out.)

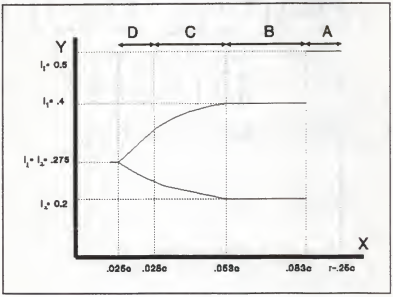

Figure 3.6

In figure 3.6, the X axis represents sunk cost F, and the Y axis represents locations of firms 1 and 2, l_1 and l_2. In region A, firm 1 is a monopolist (Lemma 3.2), locating at 0.5. In region B, there is an unconstrained duopoly equilibrium (Lemma 3.3) with firm 1 at 0.4 and firm 2 at 0.2. In region C, firm 1 is constrained by entry deterrence and firm 2 is at its unconstrained location (Lemma 3.4). In region D, both duopolists are constrained (Lemma 3.5). Finally, at $F = 0.025c$, l_1, l_2 tend to 0.275, anticipating entry of firm 3.

Section 3.5: Conclusions

In this paper we have analyzed a sequential entry equilibrium among firms with perfect foresight in a standard price discrimination model. We have analyzed equilibrium with and without entry deterring strategies for upto three firms. Where there are more firms, the analytical complexities increase, but the same general possibilities occur. The region where $F < F_{min}$, will divide into a variety of subcases. The basic idea was to illustrate the important implications of sequential entry in the spatial price discrimination framework and to discuss related issues of entry deterrence. It is open how much the specific results generalize. For example, under entry without deterrence for three firms, the equilibrium locations are for the last entrant to locate in the center with the earlier entrants located symmetrically on either side. It is open whether this type of pattern will hold when there is a larger odd number of firms. Another conjecture is that with more firms, the earlier entrants will lose much of their early mover advantage as the number of firms increases. Under entry deterrence, the results highlight issues of strategic entry deterring solutions in the context of a location model.

CHAPTER 4
ASYMMETRIC INFORMATION AND SEQUENTIAL ENTRY WITH SPATIAL PRICE DISCRIMINATION

Section 4.1: Introduction and Literature Review

This chapter examines a location model when the market grows over time, in a setting where an early entrant has better information about the market area than a potential entrant. Although this seems an important and relevant issue to examine, it has been largely overlooked in the literature. Most studies of sequential entry focus on the setting where the size of the market is fixed. [See Eaton and Lipsey, 1975; Lane. 1980; Prescott and Visscher, 1977.]

There are a number of reasons why a market can grow over time. One which is relevant for the introduction of a new product is that the market can grow as consumers take some time to be informed of its presence and reliability. Another reason for a growing market area may be increasing urbanization. Eaton and Lipsey (1979) allowed increasing demand in a spatial market with mill pricing, and analyzed the case in which an existing monopolist preempts the market by building new plants and operating them at a price less than average total cost, until market demand increases.

In a setting where the market grows over time, we analyze a duopoly model with sequential entry, where the Stackelberg leader has superior information about the market area relative to the follower. This setting is appropriate when the leader is pioneering the introduction of a new product and has devoted significant resources to market surveys, prior to entry. We discuss the question of whether the follower must invest in research or whether it can infer market information from the leader's behavior.

The basic problem and results are presented as follows. In section 4.2, the model is described. In section 4.3, we consider a one-period model. Given complete information about market type, the optimal prices and locations are derived. The analysis is extended to the case of asymmetric information about market type, and we show that the complete information equilibrium locations cannot be sustained as a Nash equilibrium. Section 4.4 analyzes the two-period model. With complete information, the location equilibria are derived for each market type. With incomplete information, it is shown that, even in the two-period model, the complete information equilibrium cannot be sustained as a Nash equilibrium.

Section 4.2: The Model

Identical consumers are uniformly distributed over a linear market. Each consumer has a perfectly inelastic demand for one unit of the homogeneous good, with reservation price r. The firms have a constant marginal cost of production (normalized to zero). Transport cost is linear in the distance transported, with c denoting the per unit transportation rate. The firms follow a policy of price discrimination, where a consumer located at x (measured from the left side of the linear market) is offered a delivered price which is a function of his location. If a consumer faces an identical price from both firms, he buys from the firm which is located closer to him. In a duopoly, the equilibrium price is the maximum of the two firms' delivered marginal costs. We assume that the reservation price is sufficiently large ($r > \theta$), so that it is not binding. Finally, the Stackelberg leader has perfect foresight regarding the behavior of the follower.

The paper has two main sections. In section 4.3, we consider a one-period model. Firm 1 acts as the Stackelberg leader, and chooses its location. Firm 2 is the follower in its location choice. Prices are chosen simultaneously after firm 2 locates, after which the market clears. The market can be of two types. If it is of length 1, it is of type L. If it is of length $\theta > 1$, it is of type H. Let l_i^j denote firm i's optimal location when the market is of type j, $i = 1, 2$, $j = L, H$, where l_1^j is measured from the left endpoint of the market and l_2^j is measured from the right endpoint of the market. We consider two alternative information sets about market type. Either market type is common knowledge or it is private information for the Stackelberg leader. In the case where market type is firm 1's private information, firm 2 knows the left endpoint. From firm 1's location, firm 2 will try to infer whether the right endpoint is at 1 or θ, where θ is common knowledge.

In section 4.4, we analyze a two-period model. In period 1, the market is of length 1. In period 2, it may remain the same, in which case it is of type L. Alternately, in period 2, it may grow from the right end, to length $\theta > 1$, in which case it is of type H. Firm 1 is the

Stackelberg leader, and chooses its location at the beginning of the first period. It has a monopoly on the market during this period. It sets its price, and the market clears. Firm 2 enters at the beginning of the second period. Price schedules are then chosen simultaneously, and the market clears. For algebraic simplicity, the discount rate is assumed to be zero. As in the one-period model, this model is also analyzed with two alternative assumptions about information regarding market type. Note that if firm 1 chooses to locate between (0, 1), then its location is denoted by l_1^j, j = L, H, with firm 2's corresponding location being l_2^j, as before. If firm 1 chooses to locate outside (0, 1), then its location is denoted by a_1^H, where a_1^H is also measured from the left side of the market. a_2^H is the corresponding location for firm 2. The different notation is useful since the profit functions are different in the two cases, leading to different expressions for optimal locations.

A price schedule (p_1^*, p_2^*) is a Nash equilibrium given locations l_i^j (or a_i^H), i = 1, 2, j = L, H, if and only if $\pi_i^*(p_i^*(x, l_1^j, l_2^j), p_j^*(x, l_1^j, l_2^j)) \geq \pi_i(p_i(x, l_1^j, l_2^j), p_j^*(x, l_1^j, l_2^j))\ \forall\ i \neq j = 1, 2$. Profit is expressed in terms of firm locations. A set of locations l_i^j (or ai^H), $i \neq j = 1, 2$, is a Stackelberg equilibrium if and only if

$$l_i^j = \arg\max_{l_i^j} \pi_i(l_i^j \mid l_k^j)\ \forall\ i \neq k = 1, 2,\ j = L, H$$

where $l_k^j = \arg\max_{l_k^j} \pi_k(l_k^j \mid l_i^j)$.

Section 4.3: One Period Model

In this section, we consider a one-period model. The market can be either of type L or of type H. Firm 1 is the Stackelberg leader in location choice. Firm 2 enters the market as the follower, after observing firm 1's location. After firm 2 chooses its location, the duopolists simultaneously choose price schedules. At this stage, the market clears. First let us assume that the market type is common knowledge. That is, both firms know exactly whether the market is of type L (that is, of length 1), or of type H (that is, of length $\theta > 1$), prior to entry. The following lemma characterizes the Nash equilibrium locations in this setup.

Lemma 4.1

> In a one-period model with complete information about market type, when the market is of type H, the Nash equilibrium locations are given by $(l_1^H, l_2^H) = ((2\theta)/5, \theta/5)$. When the market is of type L, the Nash equilibrium locations are $(l_1^L, l_2^L) = (2/5, 1/5)$.

Proof: See proof of Proposition 2.3, Chapter 2.

With sequential entry, the Stackelberg leader gets the first mover advantage, and establishes a local monopoly over a larger segment of the market (that is, the area to firm 1's left) than firm 2.

Now, we turn to the more interesting case of asymmetric information between the firms, regarding market type. Specifically, firm 1 knows the market type prior to entry. This information is not available to firm 2. We assume that firm 2 has consistent beliefs about market type, including knowledge of θ. That is, firm 2's beliefs are consistent with the complete information equilibrium locations in market types L and H respectively. Firm 2 believes that if $l_1 = l_1^H$, then market is of type H, otherwise it is of type L. Our question here is whether it is possible to infer correctly the market type from firm 1's location choice. Hence we try to see whether the complete information equilibrium can be sustained as an equilibrium with asymmetric information.

Lemma 4.2

If the market is of type H, then firm 1 has no incentive to mimic its choice in a type L market. However, if the market is of type L, then firm 1 has an incentive to mimic its choice in a type H market.

Proof: To show a) $\pi_1^H\{(2\theta)/5,\ \theta/5\} \geq \pi_1^H(2/5,\ 1/5)$ does hold while b) $\pi_1^L(2/5,\ 1/5) \geq \pi_1^L\{(2\theta)/5,\ \theta/5\}$ does not hold, $r > \theta$.

To prove step a): $\pi_1^H\{(2\theta)/5,\ \theta/5\} = \int_0^{l_1}[c(\theta - l_2 - x) - c(l_1 - x)]\ dx$

$+ \int_{l_1}^{x_H} [c(\theta - l_2 - x) - c(x - l_1)]\ dx$

where $x_H = (\theta + l_1 - l_2)/2$ ...(4.1)

$l_2 = (\theta - l_1)/3.$ ...(4.2)

$\pi_1^H\{(2\theta)/5,\ \theta/5\} = c\{(\theta - l_2)x_H - (x_H)^2 + l_1(x_H - l_1)\}$ ...(4.3)

Substituting l_1^H, (4.1) and (4.2) in (4.3) and simplyfying,

$\pi_1^H\{(2\theta)/5,\ \theta/5\} = c(\theta^2/5)$ ...(4.4)

It is easy to similarly prove that $\pi_1^H(2/5,\ 1/5) = c/5$ ...(4.5)

(4.4) and (4.5) $\Rightarrow \pi_1^H\{(2\theta)/5,\ \theta/5\} \geq \pi_1^H(2/5,\ 1/5)\ \forall\ \theta > 1$

To prove step b): $\pi_1^L(2/5,\ 1/5) \geq \pi_1^L\{(2\theta)/5,\ \theta/5)$

$\text{LHS} = \pi_1^L(2/5,\ 1/5) = c/5$ ...(4.6)

$\text{RHS} = \pi_1^L\{(2\theta)/5,\ \theta/5\}$. This will be divided into three cases.

Case i: $\theta \leq 1.67$. From figure 4.1, it implies that $x_H \leq 1$.

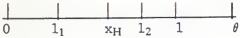

Figure 4.1

Hence $\pi_1^L\{(2\theta)/5,\ \theta/5\} = \pi_1^H\{(2\theta)/5,\ \theta/5\} = c(\theta^2)/5\ \forall\ \theta \le 1.67$.

Clearly RHS $\ge$ LHS $\forall\ \theta \le 1.67$. Therefore the separating equilibrium cannot be sustained.

Case ii: $1.67 < \theta \le 5/2$. From figure 4.2, it implies that $x_H > 1$.

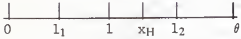

Figure 4.2

$$\text{RHS} = \pi_1^L\{(2\theta)/5,\ \theta/5\} = \int_0^{l_1}[c(\theta - l_2 - x) - c(l_1 - x)]\,dx$$

$$+ \int_{l_1}^{1}[c(\theta - l_2 - x) - c(x - l_1)]\,dx \quad \forall\ 1.67 < \theta \le 5/2.$$

Substituting l_1 and l_2 and simplyfying,

$\pi_1^L\{(2\theta)/5,\ \theta/5\} = c[-0.16\theta^2 + 1.2\theta - 1]\ \forall\ 1.67 < \theta \le 5/2.$

For step 2 to hold, LHS $\ge$ RHS.

$\Rightarrow c/5 \ge c[-0.16\theta^2 + 1.2\theta - 1]\ \forall\ 1.67 < \theta \le 5/2$

$\Rightarrow 0.16\theta^2 - 1.2\theta + 1.2 \ge 0\ \forall\ 1.67 < \theta \le 5/2$

Let $f(\theta) = 0.16\theta^2 - 1.2\theta + 1.2$

$f(0) = 1.2 > 0$. $f'(\theta) = 0.32\theta - 1.2$

$f'(0) = -1.2 < 0$. $f''(\theta) = 0.32 > 0$

$f(\theta) = 0 \Rightarrow \theta = 1.19,\ 6.31$.

Hence $\forall\ 1.19 < \theta < 6.31$, $f(\theta) < 0$. This implies that LHS < RHS $\forall\ 1.67 < \theta \le 5/2$. Hence a separating equilibrium cannot be sustained.

Case iii: $\theta > 5/2$. From figure 4.3, note that $l_1^H > 1$.

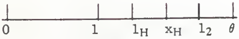

Figure 4.3

$$\text{RHS} = \pi_1^L((2\theta)/5,\ \theta/5) = \int_0^{l_1} [c(\theta - l_2 - x) - c(l_1 - x)]\ dx \quad \forall\ \theta > 5/2.$$

Simplyfying, $\pi_1^L((2\theta)/5,\ \theta/5) = c(2\theta)/5\ \forall\ \theta > 5/2$.

For step 2 to hold, LHS $\geq$ RHS $\Rightarrow c/5 \geq c(2\theta)/5 \Rightarrow \theta \leq 1/2$. Clearly step 2 does not hold for $\theta > 5/2$. From cases i - iii, we conclude that step 2 is not true for all θ. ▲

From Lemma 4.2, assuming that we are trying to sustain the complete information equilibrium locations, the informed firm has an incentive to always locate as if the market is of type H. Intuitively, if the market is actually of type H, the informed firm clearly cannot gain by mimicking the L type location $l_1^L = 2/5$, since it forfeits additional profits from the market area between 1 and θ. See figure 4.4.

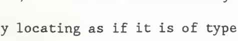

Figure 4.4

Clearly, firm 1 would be better off by signalling the truth. However, if the market is actually of type L, then firm 1 is always better off by locating as if it is of type H. In figures 4.1 to 4.3, it is shown that by mimicking type H when true market type is L, firm 1 effectively minimizes and finally eliminates (for higher θ) firm 2's share of the market. This strategy always dominates telling the truth when market is L, and sharing profits with firm 2. This result is summarized in the following proposition.

Proposition 4.1

> With consistent beliefs about market type, the one-period complete information Nash equilibrium locations cannot be sustained as a Nash equilibrium in the case where market type is firm 1's private information.

Section 4.4: Two Period Model

We now turn to a more interesting setup of the market. Specifically, the market is analyzed over two periods. Firm 1 enters at the outset of period one, and chooses its location. During this period. this firm is a monopoly in the market. It charges consumers their reservation price r, and makes monopoly profits. Firm 2 enters at the beginning of the period 2. It observes firm 1's location (which is irreversible) and chooses its location. At this stage, the duopolists simultaneously choose price schedules, after which the market clears. This means that for firm 1, a signal may have a first period cost, unlike in the one-period model; namely its first period monopoly profits.

In period 1, the market is of length 1. In period 2, the market may remain the same, in which case it is of type L. Alternatively, the market may grow in period 2, to length $\theta > 1$, in which case it is of type H. As in the single period model, we first look at Nash equilibrium locations when the market type is common knowledge. These results are summarized in Lemma 4.3 and Lemma 4.4 below.

Lemma 4.3

When the market is of type L, the complete information Nash equilibrium locations are $(l_1^L, l_2^L) = (13/28, 15/84)$.

Proof: $l_1^L = \arg\max_{l_1} \pi_1^L(l_1, l_2)$, where $l_2 = (1 - l_1)/3$

$$\text{where } \pi_1^L = \int_0^{l_1} [r - c(l_1 - x)]\, dx + \int_{l_1}^{1} [r - c(x - l_1)]\, dx$$

$$+ \int_0^{l_1} [c(1 - l_2 - x) - c(l_1 - x)]\, dx + \int_{l_1}^{x_L} [c(1 - l_2 - x) - c(x - l_1)]\, dx \quad \ldots(4.7)$$

where the first two terms equal first period monopoly profit and the last two terms equal the second period duopoly profit.

Also, $x_L = (1 + l_1 - l_2)/2$

$$\frac{\partial \pi_1^L}{\partial l_1} = 0$$

$\Rightarrow 1 - (8/3)l_1 + (2/3)(1 - l_1 - l_2) = 0$

Substituting $l_2 = (1 - l_1)/3$ and simplyfying

$\Rightarrow l_1^L = 13/28$. It follows that $l_2^L = 15/84$.

It is straightforward to show that second-order conditions are satisfied.▲

Lemma 4.4

When the market is of type H, the complete information Nash equilibrium locations are: 1) $(l_1^H, l_2^H) = \{(9 + 4\theta)/28, (24\theta - 9)/84\}$ $\forall\ \theta \le 4.75 < r$

2) $(a_1^H, a_2^H) = \{(4\theta - 9)/10, (6\theta + 9)/30\}$ $\forall\ r > \theta > 4.75$

Proof: Step 1): $l_1^H = \arg\max_{l_1} \pi_1^H(l_1, l_2)$

where $\pi_1^H = \int_0^{l_1} [r - c(l_1 - x)]\, dx + \int_{l_1}^{1} [r - c(x - l_1)]\, dx$

$$+ \int_0^{l_1} [c(\theta - l_2 - x) - c(l_1 - x)]\, dx$$

$$+ \int_{l_1}^{x_H} [c(\theta - l_2 - x) - c(x - l_1)]\, dx \qquad \ldots(4.8)$$

where $x_H = (\theta + l_1 - l_2)/2$, $l_2 = (\theta - l_1)/3$.

The first two terms equal period one monopoly profit while the last two terms equal second period duopoly profit.

$$\frac{\partial \pi_1^H}{\partial l_1^H} = 0$$

$\Rightarrow l_1^H = (9 + 4\theta)/28$. It follows $l_2^H = (24\theta - 9)/84$.

$l_1^H \leq 1 \Rightarrow \theta \leq 4.75$.

Substituting l_1^H and l_2^H into (4.8) and simplyfying, we get

$\pi_1^H(l_1^H, l_2^H) = r + c[-0.35 + 0.15\theta + 0.15\theta^2]$

It is straightforward to show that second-order conditions are satisfied.

To show step 2: $a_1^H = \arg\max_{a_1} \pi_1^H (a_1, a_2)$

where $\pi_1^H = \int_0^{1} [r - c(a_1 - x)]\, dx + \int_0^{a_1} [c(\theta - a_2 - x) - c(a_1 - x)]\, dx$

$$+ \int_{a_1}^{x_H} [c(\theta - a_2 - x) - c(x - a_1)]\, dx$$

with $a_2 = (\theta - a_1)/3$, $x_H = (\theta + a_1 - a_2)/2$.

$$\frac{\partial \pi_1^H}{\partial a_1} = 0 \Rightarrow a_1^H = (4\theta - 9)/10$$

Also $a_2^H = (6\theta + 9)/30$.

Hence $a_1^H > 1 \Rightarrow \theta > 4.75$.

Substituting a_1^H, a^{2H} into π_1^H above and simplyfying,

$$\pi_1^H(a_1^H, a^{2H}) = r + c[1.76 - 1.12\theta + 0.36\theta^2] \ \forall\ \theta > 4.75. \qquad \ldots(4.9)$$

It is straightforward to show that second-order conditions are satisfied.▲

Comparing the result in Lemma 4.3 with the corresponding locations in Lemma 4.1, the duopolists tend to move relatively closer to each other, in the two-period model. This is because, firm 1 attempts to capture the maximum profits in period 1, when it is a monopolist. The optimal monopoly location is at the center of the market. However, with foresight about firm 2's entry in period 2, firm 1 does not locate at this point. Its optimal location is to the left of its monopoly location, with foresight about the duopoly in period 2. However, relative to its location in the one-period model, it now locates closer to the center of the market, to earn period 1 monopoly profits. When the market is of type H, that is, it grows to length $\theta > 1$ at the beginning of period 2, then the first entrant has a choice. It can choose its location inside the period 1 market. This is its optimal strategy $\forall\ \theta \leq 4.75$. Alternatively, if the period 2 market is sufficiently large ($\theta > 4.75$), then firm 1 is willing to forego some of the period 1 profits in anticipation of higher profits from the much larger period 2 market. In this case, it is optimal for firm 1 to locate outside the period 1 market, in order to be relatively closer to the market in period 2.

Finally, consider the two-period model with asymmetric information. The information structure is the same as in the one-period model. Firm 1 knows the market type prior to entry. It locates at the beginning of period 1 and earns monopoly profits in the first period. At the outset of period 2, firm 2 observes firm 1's location. Firm 2 does not have any information about the market type. However it has some prior beliefs which is common knowledge. Firm 2 believes that if $l_1 = l_1^H$ (or a_1^H) then the market is of type H. Otherwise it is of type L. Hence firm 1 can signal information about market type to firm 2 through its choice of location. The next two propositions show that if, with complete information, it is optimal for firm 1 to locate inside the period 1 market, then, given that firm 2 has consistent beliefs, the complete information equilibrium cannot be sustained as a Nash equilibrium with asymmetric information.

Proposition 4.2

> With consistent beliefs about market type, the complete information equilibrium cannot be sustained as a Nash equilibrium for any θ with $\theta \leq 4.75 < r$, when the market type is firm 1's private information.

Proof: To show $\pi_1^H(l_1^H, l_2^H) \geq \pi_1^H(l_1^L, l_2^L)$ and $\pi_1^L(l_1^L, l_2^L) \geq \pi_1^L(l_1^H, l_2^H)$ cannot be simultaneously satisfied $\forall\ \theta \leq 4.75 < r$.

Step 1: To derive conditions under which $\pi_1^H(l_1^H, l_2^H) \geq \pi_1^H(l_1^L, l_2^L)$. ...(4.10)

Using (4.8) and substituting l_1^H, l_2^H from Lemma 4.4,

$\pi_1^H(l_1^H, l_2^H) = r + c[-0.35 + 0.15\theta + 0.15\theta^2]$. ...(4.11)

Also, $\pi_1^H(l_1^L, l_2^L) = \pi_1^L(l_1^L, l_2^L)$ ...(4.12)

where RHS of (4.12) = r - 0.24c (from (4.14))

Let (4.10) be true.

$\Rightarrow r + c[-0.35 + 0.15\theta + 0.15\theta^2] \geq r - 0.24c$

$\Rightarrow 0.15\theta^2 + 0.15\theta - 0.11 \geq 0.$

Let $f(\theta) = 0.15\theta^2 + 0.15\theta - 0.35$

$f(0) = -0.11 < 0$

$f'(\theta) = 0.3\theta + 0.15 \;\Rightarrow f'(0) = 0.15 > 0$

$f''(\theta) = 0.3 > 0$

$f(\theta) = 0 \;\Rightarrow \theta = -1.5,\ 0.5$. Hence, $\forall\ \theta > 0.5,\ f(\theta) > 0$.

Hence (4.10) is true $\forall\ 1 < \theta \leq 4.75$.

Step 2: To derive conditions under which $\pi_1^L(l_1^L, l_2^L) \geq \pi_1^L(l_1^H, l_2^H)$

...(4.13)

Using (4.7) and substituting l_1^L, l_2^L from Lemma 4.3,

LHS $= \pi_1^L(l_1^L, l_2^L) = r - 0.24c$ ...(4.14)

RHS $= \pi_1^L(l^{1H}, l_2^H)$. This will be analyzed in two cases.

Case i: $\theta \leq 1.83$. As is illustrated in figure 4.5, this implies that $x_H \leq 1$.

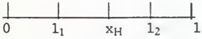

Figure 4.5

$$\pi_1^L(l^{1H}, l_2^H) = \int_0^{l_1} [r - c(l_1 - x)]\ dx + \int_{l_1}^{1} [r - c(x - l_1)]\ dx$$

$$+ \int_0^{l_1} [c(\theta - l_2 - x) - c(l_1 - x)]\ dx + \int_{l_1}^{x_H} [c(\theta - l_2 - x) - c(x - l_1)\ dx$$

...(4.15)

Integrating (4.15) and substituting l_1^H, l_2^H from Lemma 4.4,

$\pi_1^L(l_1^H, l_2^H) = r + c(0.15\theta^2 + 0.5\theta - 0.25) \; \forall \; \theta \leq 1.83.$ ...(4.16)

Let (4.13) be true.

Substituting (4.14) and (4.16),

$\Rightarrow r - 0.24c \geq r + c(0.15\theta^2 + 0.5\theta - 0.25)$

$\Rightarrow -0.15\theta^2 - 0.5\theta + 0.01 \geq 0$

Let $f(\theta) = -0.15\theta^2 - 0.5\theta + 0.01$

$f(0) = 0.01 > 0$

$f'(\theta) = -0.3\theta - 0.5$

$f'(0) = -0.5 < 0$

$f''(\theta) = -0.3 < 0$

$f(\theta) = 0 \;\Rightarrow \theta = -3.37, 0.03.$

Hence $\forall \; \theta \geq 0.03$, $f(\theta) \leq 0$. Hence (4.13) is not true for $1 < \theta \leq 1.83$.

Case ii: $1.83 < \theta \leq 4.75$. As illustrated in figure 4.6, $x_H > 1$.

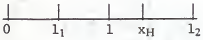

Figure 4.6

$$\pi_1^L(l_1^H, l_2^H) = \int_0^{l_1} [r - c(l_1 - x)] \, dx + \int_{l_1}^{1} [r - c(x - l_1)] \, dx$$

$$+ \int_0^{l_1} [c(\theta - l_2 - x) - c(l_1 - x)] \, dx + \int_{l_1}^{1} [c(\theta - l_2 - x) - c(x - l_1) \, dx$$

Integrating and simplyfying,

$\pi_1^L(l_1^H, l_2^H) = r + c[-0.95 + 0.81\theta - 0.04\theta^2] \; \forall \; 1.83 < \theta \leq 4.75.$

Let (4.13) be true. Then

$r - 0.24c \geq r + c[-0.95 + 0.81\theta - 0.04\theta^2]$

$\Rightarrow 0.71 - 0.81\theta + 0.04\theta^2 \geq 0$

Let $f(\theta) = 0.71 - 0.81\theta + 0.04\theta^2$

$f(0) = 0.71 > 0$

$f'(\theta) = -0.81 + 0.08\theta$

$f''(\theta) = 0.08 > 0$

$f'(0) = -0.81 < 0$

$f'(\theta) = 0 \Rightarrow \theta = 0.88,\ 19.38$

Hence, $\forall\ 0.88 < \theta < 19.38,\ f(\theta) < 0$.

$\Rightarrow$ (4.13) is not true for $1.83 < \theta \leq 4.75$.

Hence step 2 does not hold for $\theta \leq 4.75$. ▲

When the market is of type H, firm 1 has no incentive to mimic type L, since it can only lose profits from the expanded market by doing so. However, the incentive to cheat arises when the true market type is L. If firm 1 tells the truth, it earns period 1 monopoly profit and period 2 duopoly profit. However, if it cheats and pretends that the market is actually of type H, then it can push firm 2 close to the boundary of the type L market (see Figure 4.5), or even outside the market altogether (see Figure 4.6). In this case it earns period 1 monopoly profit and some profit in period 2 which dominates the duopoly profit. Clearly firm 1 has an incentive to cheat in this case.

From the above result it appears that for higher values of θ. the informed firm's incentive to cheat when the true market is of type L. will be reinforced. This intuition is borne out in the proposition below.

Proposition 4.3

When market type is firm 1's private information, then the complete information locations (a_1^H, a_2^H), (l_1^L, l_2^L) cannot be sustained as a Nash equilibrium for any θ with $r > \theta > 4.75$, when firm 2 has consistent beliefs about market type.

Proof: To show a): $\pi_1^H(a_1^H, a_2^H) \geq \pi_1^H(l_1^L, l_2^L)$

and/or b): $\pi_1^L(l_1^L, l_2^L) \geq \pi_1^L(a_1^H, a_2^H)$ $\forall\ r > \theta > 4.75$, is not true.

To prove step a:

From proof of Lemma 4.4, from (4.9) we know

LHS of a) $= r + c[1.76 - 1.12\theta + 0.36\theta^2]$

From (4.12) we have

RHS of a) $= r - 0.24c$

Suppose a) is true.

$\Rightarrow r + c[1.76 - 1.12\theta + 0.36\theta^2] \geq r - 0.24c.$

$\Rightarrow 2 - 1.12\theta + 0.36\theta^2 \geq 0$

Let $f(\theta) = 2 - 1.12\theta + 0.36\theta^2$

$f(0) = 2 > 0$

$f'(\theta) = 0.72\theta - 1.12 = 0 \Rightarrow \theta = 1.56$

$f'(0) = -1.12 < 0$

$f''(\theta) = 0.72 > 0$

$f(\theta) \neq 0$ for any real θ.

Hence $f(\theta) > 0\ \forall\ \theta > 0$. Therefore step a is true $\forall\ \theta > 4.75$.

To prove step b):

LHS of step b $= r - 0.24c$ from (4.14).

RHS of step b: $\pi_1^L(a_1^H, a_2^H) = \int_0^1 [r - c(a_1 - x)]\, dx$

$$+ \int_0^1 [c(\theta - a_2 - x) - c(a_1 - x)]\, dx$$

where the first term equals period 1 profit and the second term equals period 2 profit. Integrating and substituting a_1^H, a_2^H from Lemma 4.4,

$\pi_1^L(a_1^H, a_2^H) = r + 2c$

Suppose step b is true.

$\Rightarrow r - 0.24c \geq r + 2c$

Hence, if step b is true, then $-0.24 \geq 2$, which is a contradiction.

Hence, step b is not true. ▲

Section 4.5: Conclusions

This paper has discussed the issue of a growing spatial market, with asymmetric information among the firms. However, there is scope for relaxing some of the assumptions of the model to deal with a more general setup. For instance, here the market grows only from the right end point. It is possible to allow the market to expand from both ends. In this scenario, an analysis with two or more firms could provide qualitatively different results. Alternatively, with the same market size over two periods, a uniform increase in the density of consumers, would signify higher demand at each point. That is, at each point, transport costs are lower. Contrasting this case with the setup in this chapter, where the market grows in length, it appears that in the latter case the firms will make higher profits, since the relative distance between the consumer and the firms determines the differential between price charged and transport cost incurred in serving a customer. The other interesting change could be in the nature of beliefs held by the uninformed entrants. Here, the focus was on sustaining the complete information equilibrium locations, with asymmetric information. It may be possible to sustain a pooling or a partial pooling equilibrium.

CHAPTER 5
SUMMARY AND CONCLUSIONS

The study of a competitive spatial price discrimination model under varying assumptions about cost structure, entry and information, was the focus of this work. In chapter 2, the traditional assumption of zero or constant marginal costs of production was relaxed. With the traditional assumption, profit maximizing firms would always locate to minimize transport cost. It is shown that with nonconstant marginal costs of production, although social welfare will be maximized when they so locate, firms will not generally have an incentive to minimize transport costs alone, since its profit depends on how its location affects its rival's production cost as well as its own production and transport cost. With nonlinear production costs, each firm would want to relocate in a manner which raises its rival's marginal cost, and hence its own delivered price. The case of nonconstant marginal costs of production is then extended to a problem of output externality and the resulting pattern of location equilibria are analyzed. Finally, the chapter concludes by considering a negative externality in distance between the firms and proves that the resulting equilibrium in fact maximizes social welfare.

In chapter 3, firms are allowed to enter the market sequentially. First, we consider entry of an exogenously specified number of firms. In a duopoly, there is an asymmetric sequential location equilibrium, in direct contrast to existing results for this case. With three firms, the first two entrants locate symmetrically around the third firm. Thus in a duopoly, the earliest entrant earns a higher profit while with three firms the first two entrants earn equal profits, which are higher than that of the third firm. That is, profits are nonincreasing in order of entry and as the number of firms increases from two to three, the first entrant loses some of its first mover advantage over the second firm. The analysis is extended to make the entry process endogenous. We consider a framework where locations are chosen strategically by the incumbent firm(s) to profitably deter entry. This is possible since, in the presence of a sunk cost, the profit of a potential entrant can be made negative for certain strategic locations of incumbent firms. It is shown that in this process of endogenous sequential entry, there are ranges of sunk cost (depending on the number of incumbent firms) for which firms deter entry by locating at their unconstrained profit maximizing locations; there are ranges of sunk cost for which early entrants choose locations to create a strategic entry barrier. That is, they choose their profit maximizing locations, subject to the constraint of entry deterrence.

In the fourth chapter, the sequential entry paradigm is extended to look at entry decisions in real time. In a setting where a market may grow over time, we analyze a duopoly model with sequential entry, where the Stackelberg leader has superior information about market area, relative to the follower. In a one-period model, the Nash equilibrium prices and locations are derived and it is shown that the complete information locations cannot be sustained as a signalling equilibrium with asymmetric information. The analysis is extended to a two-period model. The informed firm enters the market first and chooses its location. It earns monopoly profits during period one. At the beginning of period two, the uninformed firm enters the market, after observing firm 1's location. It is shown that the complete information locations cannot be sustained as a signalling equilibrium, even in the two-period model.

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BIOGRAPHICAL SKETCH

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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August 1990

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